CHAPTER IX
STATISTICAL CONSIDERATIONS IN MAN-MACHINE DESIGNS

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Large portions of this handbook are statistical in nature--numbers pure and simple, or the results of analyses of statistical materials, or discussions about and suggestions for solving problems which call for the use and interpretation of statistics. A full use of this book--or any similar collection of anthropometric information--will require some acquaintance with the language of statistics and some skill in extracting from the wealth of material presented here--both explicitly and implicitly--that which is most relevant to a given problem.

Most users of this handbook already have such an acquaintance and, in varying degrees, such a skill. Nonetheless, it seems appropriate to review the statistical concepts which occur over and over again in this book and to touch on some of the statistical problems which typically confront the individuals for whom this book was prepared. This we shall do in this chapter.

The statistical concepts discussed here will be few in number and, in the main, these few will be discussed within the context of the material included in this book and the use of this material in design problems.

Initially, we shall define the basic univariate statistical measures: averages, measures of variability, and percentiles. The relationship between percentiles and mean-standard deviation combinations will be explored and tables detailing this relationship will be presented.

A brief section on the interrelationships among anthropometric variables will deal with the simple bivariate and multivariate statistics. These statistics will include the correlation coefficient as a measure of the intensity of the relationship between two variables, the regression equation as a technique for predicting or estimating the value of one dimension on the basis of one or more other anthropometric variables, and the standard error of estimate as a measure of the accuracy of such estimates. Our discussion will center on these statistics as they relate to pairs of variables, but brief comments will be included about the statistics as they apply to combinations of three, four, or more variables. An analysis of the distribution of almost 8,000 correlation coefficients from the Air Force Women's Survey of 1968 is included to provide some insight into whether--and to what extent--such coefficients tend to be large or small.

The "normal" distribution will be discussed as a mathematical model for most anthropometric data. The use of this model in the univariate case will have been anticipated in our discussion of the relationship between the mean, the standard deviation, and the percentiles. Use of the model in dealing with pairs of variables will be illustrated with artificial bivariate frequency tables, constant-probability ellipses, and problems related to the proportions of potential users disaccommodated in bivariate designs.

This chapter will conclude with brief discussions of sampling errors, "percentile men", and an example of the use of Monte Carlo methods with body size data.

The Basic Statistical Measures: One Variable at a Time

We begin with those statistics which--in vast numbers--constitute Volume II of this handbook and which appear as well throughout this volume: averages, measures of variability, and percentiles.

Averages: the mean and the median

Most common of all the statistical concepts is the notion of an average, a statistic, which is in some sense representative of an entire set of data. Of the many types of averages that have been defined, only two need concern us--the arithmetic mean and the median.

The arithmetic mean is probably the oldest and certainly the most widely used of the averages. So widespread is this use that the arithmetic mean is often not specified as such, but is referred to simply as the "mean" or the "average." Unless an average is otherwise specified, it is usually safe--particularly in the field of anthropometry--to assume it is the arithmetic mean. Similarly, the term "to average" usually signifies, to layman and professional alike, the act of computing the arithmetic mean. The unmodified term "mean" is used in the tables of this handbook, as Table 1 illustrates.

The arithmetic mean of a set of data is defined as the sum of these values divided by the number of values. Thus, for example, to determine the mean of nine values:

5,2,8,-4,4,1,5,1,5

we add them:

$$\Sigma X = 5+2+8+(-4)+4+1+5+1+5 = 27$$

We then divide the sum (27) by the number of values:

$$\text{Mean} = \bar{X} = \Sigma X/N = 27/9 = 3.0$$

TABLE 1
AN EXCERPT FROM VOLUME II: THE MAJOR UNIVARIATE STATISTICS

English Values	Mean	STD DEV	COEF OF V	-N-	1st	5th	10th	Percentiles 25th	50th	75th	90th	95th	99th
805. Stature..........													
1. Stewardesses 1971	65.45	1.91	2.92%	422		62.5	63.0	64.1	65.4	66.8	68.1	68.8	
2. US Women-D/A 1940	63.16	2.48	3.93	****		(59.1)		(61.5)		(64.8)		(67.2)	
3. WASP Pilots 1942	64.90	2.10	3.24	447		61.7	62.2	63.4	64.9	66.1	67.7	68.3	
4. AAF Nurses 1942	63.50	2.10	3.31	152		60.1	60.8	62.0	63.5	64.9	66.1	67.5	
5. WAC Separatee 1946	63.87	2.37	3.71	7563	58.5	60.0	60.8	62.2	63.8	65.5	66.9	67.8	69.5
6. WAF Basic Tr 1952	64.07	2.34	3.65	851	59.3	60.3	61.0	62.3	64.0	65.7	67.2	68.2	69.9
7. Air Force Women 1968	63.82	2.36	3.70	1905	58.9	60.0	60.7	62.1	63.8	65.4	66.9	67.8	69.5
8. WAF-Nurse Ofcrs	64.08	2.44	3.81	548	58.7	60.0	60.8	62.4	64.1	65.8	67.2	68.1	69.9
9. Enlisted WAFS/W	63.75	2.32	3.64	1216	58.9	60.1	60.8	62.1	63.7	65.3	66.8	67.7	69.4
10. Enlisted WAFS/B	63.50	2.28	3.59	131		60.0	60.5	61.7	63.4	65.2	66.7	67.5	
11. Health Exam/F 1962	63.10	2.59	4.10	3581	56.9	58.9	59.8	61.4	63.2	64.9	66.5	67.4	69.0
12. Health Ex/F 25-40	63.65	2.47	3.88	1165	58.1	59.6	60.5	62.0	63.8	65.4	66.8	67.7	69.3
13. Air Traffic Cntrl	69.56	2.50	3.59	678		65.5	66.5	67.7	69.5	71.2	72.6	73.6	
14. Army Separatee 1946	68.43	2.49	3.64	****		(64.3)		(66.8)		(70.1)		(72.5)	
15. A.A.F. Cadets 1942	69.40	2.40	3.46	2959		65.4	66.1	67.5	69.2	70.8	72.4	73.1	
16. A.A.F. Gunners 1942	67.90	2.50	3.68	583		63.4	64.5	66.2	67.9	69.5	70.9	71.7	
17. USAF Basic Tr 1952	68.54	2.61	3.81	3331	62.5	64.2	65.1	66.8	68.6	70.3	71.9	72.7	74.7
18. USAF Fly Persnl 1950	69.12	2.43	3.52	4000	63.5	65.1	66.0	67.5	69.1	70.7	72.2	73.1	75.0
19. USAF Survey 1965	69.01	2.58	3.74	3869	63.1	64.8	65.7	67.3	69.0	70.7	72.3	73.3	75.1
20. Officers 1965	69.72	2.50	3.59	549	64.4	65.5	66.3	67.9	69.8	71.5	73.0	73.8	75.1
21. Enlisted Men 1965	68.79	2.65	3.85	792	62.6	64.5	65.5	67.1	68.7	70.4	72.1	73.3	75.8
23. Basic Trainees 1965	68.93	2.55	3.70	2527	63.0	64.7	65.7	67.2	68.9	70.7	72.2	73.2	74.9
24. Navy Flyers 1964	69.94	2.33	3.33	1529	65.1	66.2	66.9	68.3	69.9	71.6	73.1	73.9	75.3
25. USAF Fly Personnel 1967	69.82	2.44	3.49	2420	64.3	65.9	66.7	68.1	69.8	71.5	73.0	73.9	75.5
26. Student Pilot 1967	69.89	2.30	3.29	505	64.8	66.2	67.0	68.3	69.8	71.5	72.9	73.8	75.3
27. Rated Pilots 1967	69.84	2.43	3.48	1187	64.1	65.9	66.8	68.2	69.8	71.5	73.0	73.9	75.5
28. SDT Navigat 1967	70.06	2.42	3.45	188		66.0	67.0	68.5	70.0	71.7	73.2	74.2	
29. RTD Navigat 1967	69.68	2.56	3.67	505	64.2	65.6	66.4	67.9	69.6	71.4	73.1	74.1	75.9
30. Army Enlisted 1965	68.71	2.60	3.78	6682	62.6	64.5	65.4	67.0	68.7	70.4	72.1	73.1	74.9
31. Navy Enlisted 1965	69.03	2.57	3.72	4095	63.2	65.0	65.8	67.3	69.0	70.7	72.4	73.4	75.2
32. Navy Divers 1972	69.38	2.36	3.40	100		65.5	66.4	67.8	69.4	71.0	72.4	73.3	

We have used X here to represent the set of individual data values and N to represent the number of data or sample size. We will use these notations often. Note also the use of Σ, the upper case Greek letter sigma, to represent the idea of "the sum of" whatever follows.

To find the mean weight of the 2,420 subjects in the 1967 USAF survey of flying personnel, we might add up all of these weights, obtaining a total of 420,088 pounds, and divide this total by the number of subjects.

$$\text{Mean Weight} = \frac{\text{Sum of Weights}}{\text{Number of Subjects}} = \frac{420{,}088}{2420} = 173.6 \text{ pounds}$$

The mean value is usually designated in tables and formulas by $\overline{X}$, M, or μ. When several sets of data are considered together, their mean values may be denoted by $\overline{X}$, $\overline{Y}$, $\overline{Z}$, or $\overline{X}_1$, $\overline{X}_2$, $\overline{X}_3$, or M_x, M_y, M_z, or some similar variation of the usual symbols. In computer printouts, notations such as M(X), M(Y) or M(1), M(2) are often used because of the limited set of symbols available on most printers.

The median is, after the arithmetic mean, the most important average. The median of a set of values is formally defined as the value in the middle when the values are arranged in numerical order, or, equivalently, the value located at a point where as many values fall below it as fall above it. Arranging the nine values we have just considered in order by size, we get:

-4,1,1,2,4,5,5,5,8.

Since the middle value is the fifth one from either end, the median of the group is 4.

The median is also the 50th percentile--a concept we shall soon define --and is listed among the percentiles in Volume II and throughout this handbook. From Table 1, we note that the median stature of the stewardesses was 65.4 inches. The comments we shall make about the computation of percentiles apply equally to the computation of the median.

For most anthropometric data--and for all types of data for which the normal distribution is a reasonable model--the mean and the median tend to be almost equal. The median of the USAF '67 flying personnel weights is 172.4 pounds, a trifle lower than the 173.6 pound value we obtained for the mean. This difference of scarcely more than a pound probably represents the most significant difference to be found among our mean/median data for these men. The mean and the median for the total height (stature) of these fliers--statistically a much more typical set of data--were 69.82 inches and 69.78 inches respectively. Here the mean is a mere twenty-fifth of an inch larger than the median. Other mean/median comparisons can be made using the values in Table 1. There are, it is true, a few anthropometric variables for which this level of close agreement between the mean and the median does not exist. This lack of agreement will be most substantial for age and skinfold measures, variables not directly related to basic design problems. For most sets of data, we have reported the mean and, as the 50th percentile, the median.

In addition to the mean and the median, there are two averages which, it can be argued, are more logically related to design problems: the mode and the mid-range value. The mode is defined as the most frequently occurring value in a set of data and the mid-range as the average of the maximum and minimum values. We have included neither of these averages here for two reasons. Both statistics, when computed on large sets of continuous data; are highly dependent on the precise method of computation and editing and are highly sensitive to minor variations in measurement techniques and sample selection. In addition, whenever the normal probability model is appropriate, the mode, the mid-range, the mean and the median are all theoretically equal. If, then, all four of these averages are, in theory, equal, our choice among them is logically the one we can determine most accurately from a sample of a given size. On this basis, the arithmetic mean is clearly the preferred statistic.

None of these averages--considered by itself--has great usefulness in design problems. It is true, of course, that there are more men of average height than of any other particular height, but it is equally true that most men are shorter than average or taller than average, some of them by small amounts and others by considerable ones. Along with our averages, we need statistical measures which measure and describe the variations, large and small, up and down from the average value. These are discussed next.

Measures of Variability: the Standard Deviation and the Coefficient of Variation

A pioneer in the field of statistics, Sir Francis Galton, wrote years ago that "it is difficult to understand why statisticians commonly limit their interests to averages. Their souls seem as dull to the charm of variety as that of a native of one of our flat English counties whose retrospect of Switzerland was that, if its mountains could be thrown into its lakes, two nuisances could be got rid of at once." Basic to virtually all design problems is the fact that mankind is far more like Switzerland than a flat English county, and that, whatever the charms of variety may be, we need statistics to quantify this variety.

The standard deviation is virtually the sole measure of variability of concern to us. The coefficient of variation is also of considerable importance, but this statistic, as we shall see, is simply a restatement of the standard deviation as a percent of the mean.

The standard deviation for a set of data can be obtained by the following sequence of steps:

a. compute the mean value: $(\overline{X})$;
b. compute the deviation of each value from the mean: $(X-\overline{X})$;

c. square these deviations: $(X-\overline{X})^2$;
d. obtain the mean of these squares: $\Sigma(X-\overline{X})^2/N$;
e. compute the square root of this quantity.

The value at this last step is the standard deviation. Stated as a formula*:

$$\text{Standard deviation} = \sqrt{\Sigma(X-\overline{X})^2/N}$$

To compute the standard deviation of the nine values we have just averaged (5,2,8,-4,4,1,5,1,5), we follow this sequence of steps:

a. we already have $\overline{X} = 3$
b. the deviations are 2,-1,5,-7,1,-2,2,-2,2
c. the squared deviations are 4,1,25,49,1,4,4,4,4
d. their mean value is $(4+1+\cdots+4)/9 = 96/9 = 10.7$
e. and the square root of $10.7 = 3.26$

The sequence of steps usually used to compute the standard deviation differs from that just described, but is mathematically equivalent and gives identical results.

The standard deviation is commonly denoted either by the initials SD or by σ, the lower case Greek letter sigma, with, if necessary, suitable subscripts. The use of σ is sufficiently general so that the word "sigma" itself is sometimes used to denote the standard deviation. In Table 1, the standard deviation is the second of the statistics, listed in the columns headed "STD DEV".

The way the standard deviation relates to the distribution of a set of data is illustrated by the four graphs in Figure 1. The first of these graphs represents the statures (total heights) of all the women measured in the Air Force Women's survey of 1968. The mean of these heights is 63.82 inches and the standard deviation is 2.36 inches.

The three other graphs also represent the statures of women measured in this survey, but correspond to subgroups chosen on the basis of each woman being of average value in a second measurement: weight, sitting height, or cervicale height. Because of the relationships between stature and the other measurements, the women in these subgroups are less variable in their heights and the standard deviations decrease progressively as we go from curve (a) to curve (d). The standard deviation for the total series was 2.36 inches as we have already noted; the other standard deviations are, in order, 2.00 inches, 1.42 inches, and 0.50 inches. The mean value in each case remains 63.82 inches.

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*Sometimes N-1 or N-1.5 is used in place of N in this formula. When the standard deviation is considered as a descriptive statistic, the proper divisor is N. When the value of N is large, it makes little difference which divisor is used. The formula given here was used in computing the standard deviation for most major sets of data in Volume II.

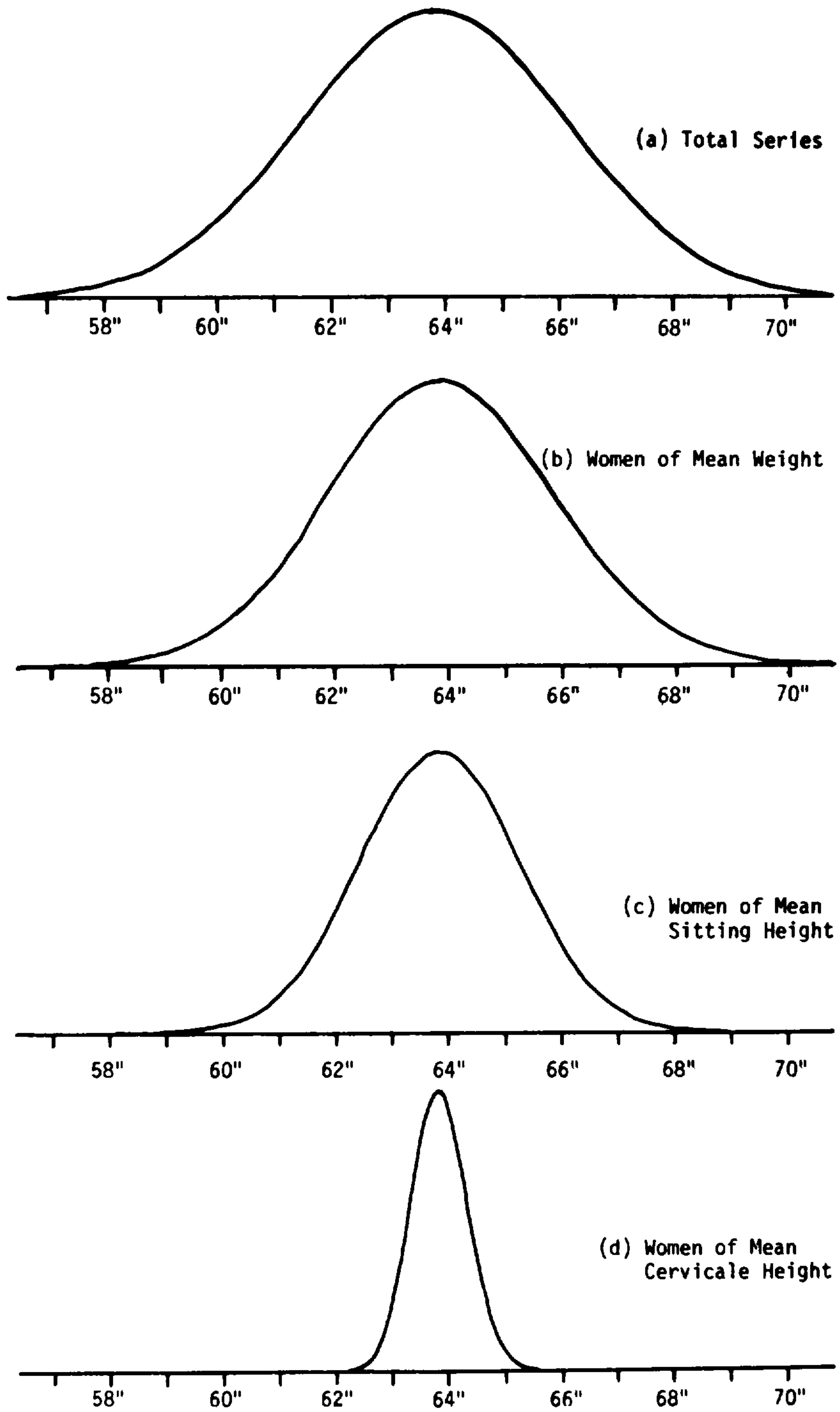

Figure 1. Distribution of stature measurements (AFW'68 data).

The range in statures in graph (a) would appear to be from about 7 inches below the mean (56.8 inches) to about 7 inches above it (70.8 inches). Graph (b) is a little narrower than graph (a). Here the range would seem to be about 6 inches up and down from the mean value (57.8 inches to 69.8 inches). Graph (c) is in turn still narrower--only slightly more than half as wide as the first graph. Here the range seems roughly about $X \pm 4.2$ inches or from 59.6 inches to 68.0 inches. Finally, the last graph, little more than 20% as wide as the first one, shows a range of statures from about 62.3 inches to 65.3 inches.

The ranges suggested by these graphs are, in each case, from approximately three standard deviations below the mean ($\bar{X}$ - 3 SD) to three standard deviations above it ($\bar{X}$ + 3 SD). Other important points on the distribution of a set of anthropometric data can be located, at least approximately, by adding or subtracting a multiple of the standard deviation to the mean value. In particular, it is worth noting (see also Figure 2) that:

about 2/5 of a set of data fall between $\bar{X}$-0.5 SD and $\bar{X}$+0.5 SD
about 2/3 of a set of data fall between $\bar{X}$-1.0 SD and $\bar{X}$+1.0 SD
about 87% of a set of data fall between $\bar{X}$-1.5 SD and $\bar{X}$+1.5 SD
about 95% of a set of data fall between $\bar{X}$-2.0 SD and $\bar{X}$+2.0 SD
almost all of a set of data fall between $\bar{X}$-3.0 SD and $\bar{X}$+3.0 SD.

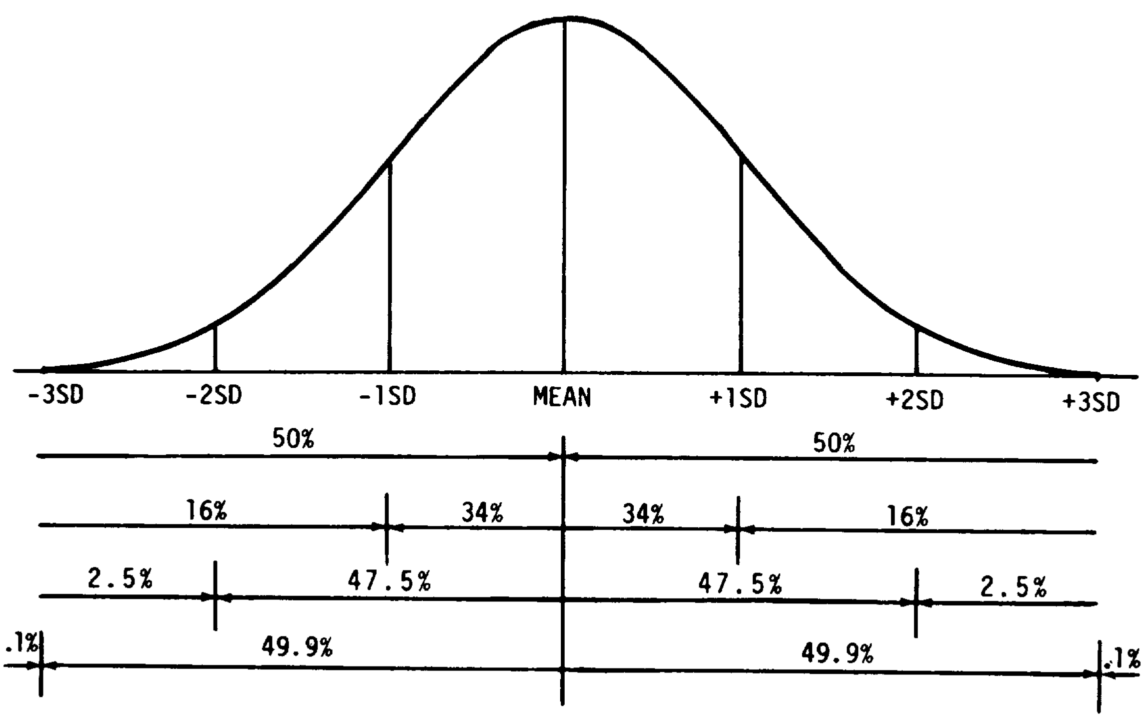

Figure 2. Areas under the normal curve.

These figures can be restated in several ways. One could say, for example, that about one-third of the data will fall in the range from the mean to the mean plus a standard deviation and that about one-sixth of the data will exceed the mean plus a standard deviation.

In Table 2, we have listed for more or less normally distributed data the approximate percentages which will fall into ranges which are based on the mean ($\bar{X}$) and various multiples of the standard deviations (K•SD).* Here we may note, for example, that if K = 0.5, then:

about 31% of a set of such data fall below $\bar{X}$ - K•SD	(Column A)
about 31% fall above $\bar{X}$ + K•SD	(Column A)
about 38% fall between $\bar{X}$ - K•SD and $\bar{X}$ + K•SD	(Column B)
about 69% fall below $\bar{X}$ + K•SD	(Column C)

To illustrate one typical use of a table such as Table 2, we can estimate the proportion of USAF flying personnel who are taller than 6'1". Our best data for these men are those from the USAF '67 flying personnel survey. From Table 1 (or Volume II) we find that the appropriate statistics are these:

Mean stature: 69.82"; standard deviation: 2.44".

Using these figures, we next determine how far 6'1" is above the mean in standard deviation units:

$$K = \frac{6'1''-\bar{X}}{SD} = \frac{73.00-69.82}{2.44} = \frac{3.18}{2.44} = 1.30$$

Column A in Table 2 gives a value of 9.7% for K=1.30, from which we may conclude that about 10% of the Air Force's male fliers are 6'1" tall or taller.

We can similarly estimate the proportion of Air Force women shorter than 5' 1". From Table 1, we obtain the relevant statistics from the survey of such women made in 1968:

mean stature: 63.82"; standard deviation: 2.36".

On the basis of these statistics, 5' 1" is 2.82" or 1.19 standard deviations below the mean. Entering Table 2 with the value K = 1.2, we get 11.5% as the approximate number of these women shorter than 5 feet. Since there are virtually no Air Force women taller than 6'1" and virtually no flying personnel shorter than 5 feet, we are in a position to conclude that a design range for statures from 61" to 73" would include roughly 90% of both the USAF flying personnel and USAF women.

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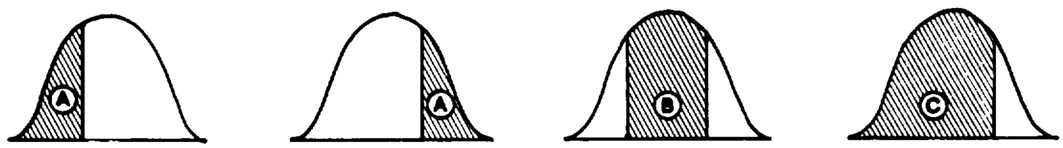
*More detailed versions of Table 2 (and Table 5) are available in Abramowitz and Stegun (1964).

TABLE 2

APPROXIMATE PROPORTIONS OF DATA FALLING INTO INTERVALS BASED ON MEAN $\pm$K STANDARD DEVIATIONS



$\bar{X}$-KSD $\bar{X}$+KSD $\bar{X}$-KSD $\bar{X}$+KSD $\bar{X}$+KSD

K	A	B	C
0.0	50.0%	0.0%	50.0%
0.1	46.0%	8.0%	54.0%
0.2	42.1%	15.8%	57.9%
0.3	38.2%	23.6%	61.8%
0.4	34.5%	31.1%	65.5%
0.5	30.9%	38.3%	69.1%
0.6	27.4%	45.1%	72.6%
0.7	24.2%	51.6%	75.8%
0.8	21.2%	57.6%	78.8%
0.9	18.4%	63.2%	81.6%
1.0	15.9%	68.3%	84.1%
1.1	13.6%	72.9%	86.4%
1.2	11.5%	77.0%	88.5%
1.3	9.7%	80.6%	90.3%
1.4	8.1%	83.8%	91.9%
1.5	6.7%	86.6%	93.3%
1.6	5.5%	89.0%	94.5%
1.7	4.5%	91.1%	95.5%
1.8	3.6%	92.8%	96.4%
1.9	2.9%	94.3%	97.1%
2.0	2.3%	95.4%	97.7%
2.1	1.8%	96.4%	98.2%
2.2	1.4%	97.2%	98.6%
2.3	1.1%	97.8%	98.9%
2.4	0.8%	98.4%	99.2%
2.5	0.6%	98.8%	99.4%
2.6	0.5%	99.1%	99.5%
2.7	0.3%	99.3%	99.7%
2.8	0.3%	99.5%	99.7%
2.9	0.2%	99.6%	99.8%
3.0	0.1%	99.7%	99.9%

An important restatement of the standard deviation is known as the coefficient of variation. This statistic, often designated by the letter V, is the standard deviation expressed as a percentage of the mean value:

$$V = \frac{SD}{\bar{X}} \cdot 100\% \quad \text{or} \quad (SD/\bar{X}) \cdot 100\%$$

Thus, the coefficient of variation of the statures measured in the USAF '67 flying personnel survey, based on the statistics just used, is

$$V = \frac{2.44''}{69.82''} \cdot 100\% = 3.49\%$$

The coefficients of variation are presented for all sets of data in Volume II*. They are designated there, as can be seen from Table 1, by "COEF OF V."

The importance of the coefficient of variation for body size data is that this statistic tends to have roughly the same value for anatomically similar measurements. A few values, based on the 1968 Air Force Women's survey and the USAF '67 and USAF '50 flying personnel surveys, are shown in Table 3. Weight usually has a coefficient of variation of 10%-15% for military samples, skinfold measures have values in the 30% to 50% range, but most measurements have considerably smaller values. The major head measurements have among the lowest values of V, usually in the 2.5% to 3.5% range. Heights and long bone measurements have coefficients of variation in the 3.5% to 5.0% range. Major circumferences, breadths, and depths have values usually falling between 5% and 10%. Within these broad categories, the smaller the measurement, the larger the coefficient of variation is likely to be, in part because the smaller the measurement, the relatively greater the measurement error. The more closely a measurement is related to the bony structure of the body, the smaller the value of V. Thus, for example, the values of V in Table 3 for shoulder circumference (5.0-5.2%) are only about 60% as great as those for waist circumference (8.2-9.3%). Small measurements not based on bony landmarks are particularly prone to large coefficients of variation.

There are a few standard anthropometric measures which do not correspond to a single anatomic entity as much as they represent the difference between two such entities. For such measurements, the coefficient of variation is likely to be quite high. A major example of such a measurement is elbow-rest height--the distance from the underside of the elbow to the

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*The coefficient of variation is clearly independent of the units in which a measurement is expressed. However, there are occasional minor differences in Volume II between the values of V given with the metric data and those given with the English values. This is because V was computed in each case from the values of $\bar{X}$ and SD exactly as they are listed.

TABLE 3
COEFFICIENTS OF VARIATION BY MEASUREMENT TYPE

a) Major Head Measurements (2.5%-4.0%)

	1967 Flying Personnel	Air Force Women	1950 Flying Personnel
Head circumference	2.5%	3.0%	2.7%
Head length	3.4%	3.7%	3.3%
Head breadth	3.5%	4.1%	3.4%

b) Major Heights and Long Bone Lengths (3.5%-5.5%)

	1967 Flying Personnel	Air Force Women	1950 Flying Personnel
Stature	3.5%	3.8%	3.6%
Acromial height	4.0%	4.2%	4.0%
Cervicale height	3.9%	4.0%	3.9%
Chest height	4.1%	4.5%	4.1%
Waist height	4.5%	4.5%	4.3%
Crotch height	4.9%	5.5%	5.2%
Sitting height	3.5%	3.8%	3.6%
Knee height sitting	4.5%	-	4.6%
Sleeve length	3.9%	4.2%	4.5%

c) "Bony" Circumferences (5.0%-6.5%)

	1967 Flying Personnel	Air Force Women	1950 Flying Personnel
Shoulder circumference	5.0%	5.2%	5.2%
Ball of foot circumference	5.0%	-	5.0%
Knee circumference	5.4%	6.3%	5.8%
Wrist circumference	5.2%	4.8%	5.3%
Buttock circumference	5.6%	6.0%/6.4%	6.0%
Chest circumference	6.5%	6.4%	6.2%

d) "Fleshy" Circumferences, Breadths, Depths (6.5%-10.0%)

	1967 Flying Personnel	Air Force Women	1950 Flying Personnel
Waist circumference	8.5%	8.2%	9.3%
Biceps circumference (relaxed)	7.6%	9.0%	7.9%
Thigh circumference	7.6%	7.7%	7.6%
Calf circumference	6.2%	6.6%	6.5%
Buttock depth	8.6%	8.5%	9.2%
Chest breadth	6.5%	6.9%	6.6%
Chest depth	7.9%	8.2%	8.2%

e) Weight (10%-15%)

	1967 Flying Personnel	Air Force Women	1950 Flying Personnel
Weight	12.4%	13.1%	12.8%

f) Skinfolds (30%-50%)

	1967 Flying Personnel	Air Force Women	1950 Flying Personnel
Triceps	40.2%	28.5%	-
Subscapular	38.7%	37.3%	-
Juxtánipple	49.3%	-	-

sitting surface. This measurement is basically the difference between sitting shoulder height and shoulder-elbow length and, not surprisingly, usually has a coefficient of variation in excess of 10% even though it is classified as a "length."

In addition there are a few measurements for which the coefficient of variation is not an appropriate statistic. Primarily, these are measurements for which the zero value is arbitrary. An example, illustrated in Figure 3, relates to the inclination of a line joining the center of the earhole and the outer corner of the eye. We could measure the angle this line makes with a horizontal axis (θ) or its angle with a vertical axis (ϕ). Both (θ) and (ϕ) will contain the same information, be equally valid and useful, and have the same standard deviation. However, since the mean value of the first will be about 10° and of the second about 80°, the coefficient of the first will be about eight times as large as the first.

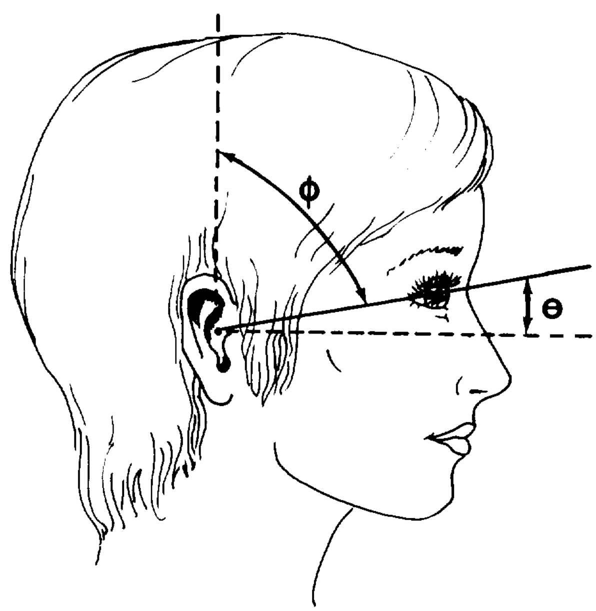

Figure 3. Measurement with an arbitrary zero value.

Other measures of variability are occasionally used: the range, the mean deviation, the probable deviation, the semi-interquartile range, and so forth. The range is simply the difference between the largest and smallest values in a set of data. The mean deviation is the average of the absolute values of the deviations from the mean (sometimes from the median). The probable deviation is about two-thirds as large as the standard deviation; it was defined so that 50% of a set of data would fall within a probable deviation of the mean. The semi-interquartile range is half the distance from the 25th percentile (soon to be defined) and the 75th percentile.

Of these, only the range is likely to be encountered in compilations of anthropometric data. The range is obviously an easily computed and easily understood statistic. Unfortunately, except as a purely descriptive statistic, it is a notoriously poor one because it is dependent on sample size, because its sampling error decreases at no more than a snail's pace as the sample size increases, because it is completely dependent on the two most atypical and most probably erroneous individual values in a set of data, and because, when computed from edited data, it is highly dependent on the subjective judgment of the editor. Range values have not been included in Volume II.

The Percentiles

The class of statistics which are most closely related to design problems are the percentiles and other so-called measures of position.

The definition of the percentiles is fairly simple. For any set of data--the weights of a group of pilots, for example--the first percentile is a value which is, on the one hand, greater than the weights of each of the lightest 1% of the pilots and is, on the other hand, less than the weights of each of the heaviest 99% of these men. Similarly, the second %ile is greater than each of the lightest 2% and less than each of the heaviest 98%. Whatever the value of K--from 1 to 99--the K-th percentile is a value greater than each of the smallest K% of the weights and less than the largest (100-K)%. The 50th percentile, which we encountered among the averages as the median, is a value dividing a set of data into two groups containing the smallest and largest 50% of the values.

The role of percentiles in many types of design problems is to provide a basis for judging the proportion of a group of individuals who exceed --or fall below--some possible design limit. There are, naturally, 99 percentiles, from the 1st to the 99th, although even the most complete computations of body size data are usually limited to the 1st, 2nd, 3rd, 5th, 10th, ..., 90th, 95th, 97th, 98th, and 99th. Space constraints have limited those listed in Volume II to the 9 most important of these as they appear in Table 1. Those omitted--mostly percentiles between the 25th and the 75th--are rarely, if ever, used in design problems and can, as we shall see, be easily approximated if they are needed. A few of the percentiles in addition to the median have other names. In particular, the 25th and 75th percentiles are the 1st and 3rd quartiles (the median is the 2nd quartile); and the 10th, 20th, etc. percentiles are also known as the 1st, 2nd, etc. deciles.

The computation of the percentiles is not quite as simple as our definition would suggest. The basic problem is that, in general, there are no values which satisfy the definition. A strict reading of the definition says that the 1st percentile is a weight such that 1% of the 2,420 flyers (or 24.2) are lighter and 99% (or 2,395.8) are heavier. One problem is that we are limited to integer numbers of men; we can count off 24 or 25 men, but not 24.2. A second problem is that we can't really arrange all 2,420 men in order of their weights; all these men undoutedly have different weights, but they don't all have different recorded weights. In practice, we rely on computational methods based on the spirit, rather than the precise letter, of the definition.

One useful method of computing percentiles is based on the special graph paper shown in Figure 4. This graph paper has been designed so that we get points which fall on a straight line if we plot the cumulative frequencies of a perfect normal distribution. Plots of real data for body size dimensions on this type of graph paper usually consist of points which can be fitted by a smooth curve which, at least in the mid-range, is almost linear.

To illustrate the process, we have provided in Table 4 the frequency table for U.S. Navy pilots' statures. In Figure 4, the cumulative frequencies are plotted against the upper limits of the intervals in this table. We have drawn on this figure a smooth curve passing close to, but not always through, the plotted points. The percentiles are ultimately read from this curve. Thus, for example, we note that the 5th percentile here is 168.3 cm, the 10th percentile is 170.0 cm, etc. The computational procedure not only circumvents the problems we have discussed, but also tends to minimize the irregularities from which data from finite samples always suffer. In Figure 5 we have plotted the same points on conventional graph paper to illustrate the differences in the graphs which the two types of paper provide.

Percentiles for the major series of data included in this handbook were computed using a method similar to this graphic one but one designed for use on a computer in order to reduce the labor involved and to provide more objective results. Full details of this method, including the computer program, are given in Anthropometry of Air Force Women by Clauser and his associates. Every set of percentiles appearing in Volume II which includes the 1st and 99th percentiles were computed using this computer program. Percentiles for a few small series of data included in Volume II were also computed by this method, but the extreme percentiles are not listed because of sample size. Details of the calculation of most of the other percentiles listed in Volume II, unfortunately, have never been published.

Our earlier discussion of the mean and the standard deviation came close to establishing--for more or less normally distributed data--a relationship between the standard deviation and the percentiles. Table 5 makes this relationship more explicit by indicating for each percentile its distance in standard deviations above or below the mean. The table indicates, for example, that the 5th and 95th percentiles are, approximately, 1.645 standard deviations below and above the mean; these percentiles for USAF '67 flying personnel statures can thus be approximated as 69.82-1.645•2.44=65.8" and 69.82+1.645•2.44=73.8", values not very different from those shown in Table 1 (65.9" and 73.8").

Table 5 points up an important fact about percentiles: the difference between consecutive percentiles increases substantially as one goes from

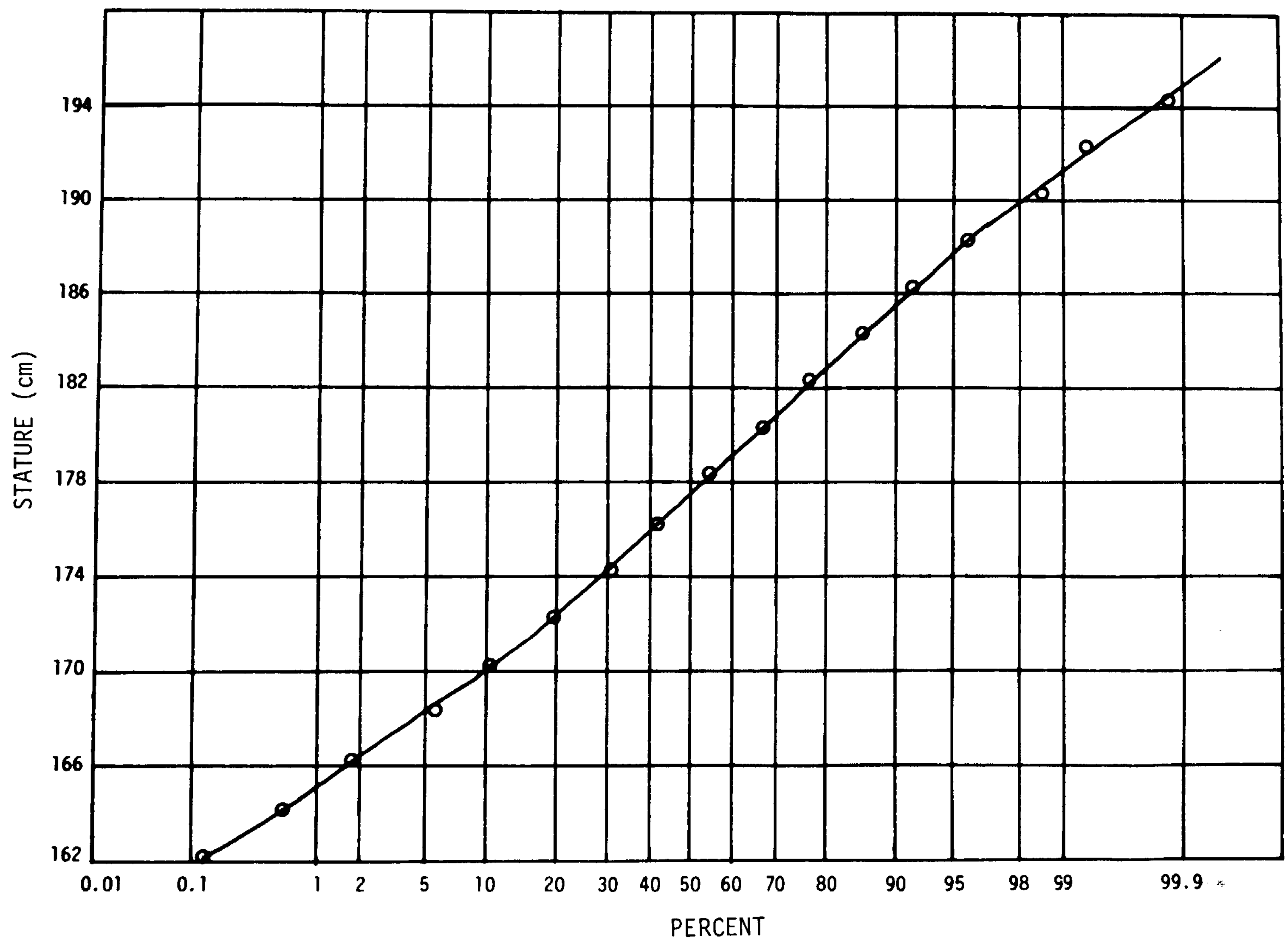

Figure 4. Computation of percentiles using normal probability graph paper.

TABLE 4
FREQUENCY TABLE FOR U.S. NAVY PILOTS' STATURES

Value	F	Cum F	Cum F %
194.25-196.25	2	1529	100.00
192.25-194.25	8	1527	99.87
190.25-192.25	11	1519	99.35
188.25-190.25	40	1508	98.63
186.25-188.25	62	1468	96.01
184.25-186.25	93	1406	91.96
182.25-184.25	129	1313	85.87
180.25-182.25	157	1184	77.44
178.25-180.25	192	1027	67.17
176.25-178.25	191	835	54.61
174.25-176.25	180	644	42.12
172.25-174.25	173	464	30.35
170.25-172.25	133	291	19.03
168.25-170.25	74	158	10.33
166.25-168.25	58	84	5.49
164.25-166.25	18	26	1.70
162.25-164.25	6	8	0.52
160.25-162.25	2	2	0.13

TABLE 5
PERCENTILE-STANDARD DEVIATION RELATIONSHIPS

Percentile		Percentile	Percentile		Percentile
1st	M* ± 2.326 SD**	99th	26th	M ± 0.643 SD	74th
2nd	M ± 2.054 SD	98th	27th	M ± 0.613 SD	73rd
3rd	M ± 1.881 SD	97th	28th	M ± 0.583 SD	72nd
4th	M ± 1.751 SD	96th	29th	M ± 0.553 SD	71st
5th	M ± 1.645 SD	95th	30th	M ± 0.524 SD	70th
6th	M ± 1.555 SD	94th	31st	M ± 0.496 SD	69th
7th	M ± 1.476 SD	93rd	32nd	M ± 0.468 SD	68th
8th	M ± 1.405 SD	92nd	33rd	M ± 0.440 SD	67th
9th	M ± 1.341 SD	91st	34th	M ± 0.412 SD	66th
10th	M ± 1.282 SD	90th	35th	M ± 0.385 SD	65th
11th	M ± 1.227 SD	89th	36th	M ± 0.358 SD	64th
12th	M ± 1.175 SD	88th	37th	M ± 0.332 SD	63rd
13th	M ± 1.126 SD	87th	38th	M ± 0.305 SD	62nd
14th	M ± 1.080 SD	86th	39th	M ± 0.279 SD	61st
15th	M ± 1.036 SD	85th	40th	M ± 0.253 SD	60th
16th	M ± 0.994 SD	84th	41st	M ± 0.228 SD	59th
17th	M ± 0.954 SD	83rd	42nd	M ± 0.202 SD	58th
18th	M ± 0.915 SD	82nd	43rd	M ± 0.176 SD	57th
19th	M ± 0.878 SD	81st	44th	M ± 0.151 SD	56th
20th	M ± 0.842 SD	80th	45th	M ± 0.126 SD	55th
21st	M ± 0.806 SD	79th	46th	M ± 0.100 SD	54th
22nd	M ± 0.772 SD	78th	47th	M ± 0.075 SD	53rd
23rd	M ± 0.739 SD	77th	48th	M ± 0.050 SD	52nd
24th	M ± 0.706 SD	76th	49th	M ± 0.025 SD	51st
25th	M ± 0.674 SD	75th	50th =	M	

* Mean
** Standard Deviation

the center out to either end of the range. To emphasize this point, in Table 6 we have taken the difference between the 50th and the 51st percentiles as a "mid-range design unit" and have tabulated, in terms of this unit, the increases in the width of a design which would be required in order that it cover an additional one percent of the population. This cost rises slowly over the middle of the range; to go from 75th to 76th percentile requires an increase only about 1.3 times as large as was required to go from the 50th to the 51st. Not until we are almost at the 90th percentile does an increase of one percentile value cost twice the mid-range unit but from there on the cost increases rapidly. To include the one percent of the population between the 98th and 99th percentiles will require an increase of almost 11 mid-range design units. We can be confident that the top one percent of the values will be spread over an exceedingly wide range, but it is unrealistic to expect accurate estimates of just how wide.

Measures of Symmetry and Kurtosis

Measures of symmetry (β_1) and kurtosis (β_2) are sometimes given in reports of anthropometric surveys. Since these statistics are usually close to the normal distribution values of 0.0 and 3.0 for body measurements of interest to the design engineer, we have not included them in this handbook. The value of β_1 (sometimes spelled out as veta, corresponding to the Greek pronunciation, other times as beta) is based on the cubes of the differences between the data and their mean. Positive values of β are suggestive of a pattern in which data are distributed at greater distances above the mean than they are below it. The value of β_2 is based on the fourth power of these differences and normally relates to the degree of peakedness of the distribution of the data.

The Interrelationship Among Anthropometric Measures

Tall men tend to have long arms, short men tend to be below average in hip breadth. Men with long faces, on the other hand, are almost as likely to have narrow faces as they are to have wide ones. All anthropometric measures are to one degree or another statistically related to each other; the nature and degree of these relationships are often matters of substantial importance in the design of equipment, workspace, and clothing.

In Figure 6 we have illustrated examples of four rather different degrees of relationship:

a. the almost perfect relationship between stature and stature maximum;
b. the less close but still quite substantial relationship between weight and shoulder circumference;
c. the modest relationship between stature and weight;
d. the almost negligible relationship between lip length and face length (menton-sellion length).

TABLE 6

COST OF ACCOMMODATING ADDITIONAL PERCENTAGES OF A USER-POPULATION IN MID-RANGE UNITS

Population Percentage	Cost in Mid-Range Units*
50th to 51st	1.00 unit
60th to 61st	1.04 units
70th to 71st	1.16 units
75th to 76th	1.27 units
80th to 81st	1.45 units
85th to 86th	1.75 units
90th to 91st	2.36 units
91st to 92nd	2.56 units
92nd to 93rd	2.82 units
93rd to 94th	3.15 units
94th to 95th	3.59 units
95th to 96th	4.22 units
96th to 97th	5.18 units
97th to 98th	6.88 units
98th to 99th	10.86 units
(99th to 99.5th)	19.88 units/percent
(99.5th to 99.9th)	51.24 units/percent

*i.e., the width of the interval required for a particular percent expressed as multiples of the width of a similar interval near the center of the distribution.

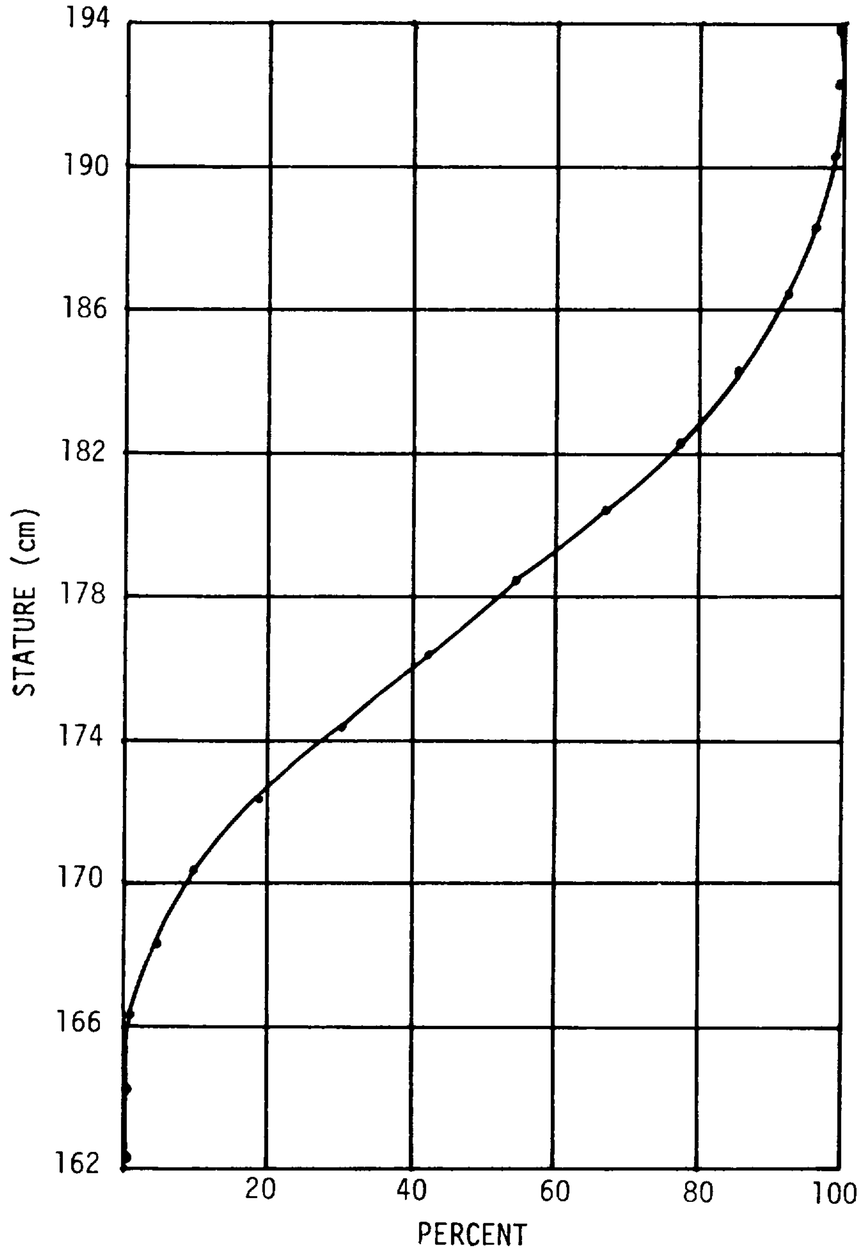

Figure 5. Cumulative frequencies--U.S. Navy Flyers'64 statures--on rectangular graph paper.

STATURE AND STATURE, MAXIMUM

STATURE, MAXIMUM (columns); STATURE (rows)

STATURE	145 .25	147 .25	149 .25	151 .25	153 .25	155 .25	157 .25	159 .25	161 .25	163 .25	165 .25	167 .25	169 .25	171 .25	173 .25	175 .25	177 .25	179 .25	181 .25	183 .25	TOT ALS
183.25																				1	1
181.25																			1		1
179.25																		8	1		9
177.25																	9	5			14
175.25																10	3				13
173.25														1	41	11					53
171.25														61	21						82
169.25												1	83	41							125
167.25												113	74								187
165.25										1	125	57									183
163.25										164	77										241
161.25								2	183	75											260
159.25								146	65												211
157.25							132	58													190
155.25						101	59														160
153.25					58	29	1														88
151.25				42	18																60
149.25			13	4																	17
147.25		5	3																		8
145.25	2																				2
Totals	2	5	16	46	76	130	192	206	248	240	202	171	157	103	62	21	12	13	2	1	1905

Summary Statistics

	Mean	Std Dev	Regression Equations	SE-Est
Y-Stature	162.10	6.00	0.995X + 0.169	0.38
X-Stature, Maximum	162.75	6.02	1.001Y + 0.482	0.38

A. An exceedingly close relationship: correlation coefficient = 0.998

WEIGHT AND SHOULDER CIRCUMFERENCE

SHOULDER CIRCUMFERENCE (columns); WEIGHT (rows)

WEIGHT	87 .25	89 .25	91 .25	93 .25	95 .25	97 .25	99 .25	101 .25	103 .25	105 .25	107 .25	109 .25	111 .25	113 .25	115 .25	117 .25	119 .25	121 .25	TOT ALS
200.00														1				1	2
195.00																1	1		2
190.00													1			1			2
185.00												1		1	3		2		7
180.00											1			1	1				3
175.00									1			1	1	1	2	1			7
170.00									1		2	5	1	2	2	1			14
165.00									2	2	4	6	1		2				17
160.00								1	1	3	2	15	1	2	1	3	1		30
155.00							1	1	5	10	10	13	6	3					49
150.00					1			3	19	13	17	12	5	2					72
145.00						2	4	19	28	23	29	12	4	1					122
140.00					3	5	14	24	28	34	22	8	3	1					142
135.00				2	6	5	22	56	44	28	16	1	1						181
130.00				1	9	28	52	58	48	27	7	1							231
125.00			1	1	24	44	72	56	28	12	3								241
120.00				14	29	53	74	32	16	3									221
115.00			5	21	48	60	54	15	5	1									209
110.00		2	7	25	49	47	14	8											152
105.00		4	11	37	29	25	6	1											113
100.00		6	15	21	17	2													61
95.00	1	2	10	6	3														22
90.00			2	2															4
85.00	1																		1
Totals	2	14	51	130	218	271	313	274	226	156	113	75	24	15	11	7	4	1	1905

Summary Statistics

	Mean	Std Dev	Regression Equations	SE-Est
Y-Weight	127.28	16.59	2.695X - 143.330	9.13
X-Shoulder Circ	100.41	5.14	0.269Y + 67.447	2.83

B. A close relationship: correlation coefficient = 0.835

Figure 6. Bivariate frequency tables illustrating interrelationships of anthropometric data (from Clauser et al. 1972).

STATURE

WEIGHT	145 .25	147 .25	149 .25	151 .25	153 .25	155 .25	157 .25	159 .25	161 .25	163 .25	165 .25	167 .25	169 .25	171 .25	173 .25	175 .25	177 .25	179 .25	181 .25	183 .25	TOTALS
200.00										1							1				2
195.00									1					1							2
190.00										1		1									2
185.00										2	2			3							7
180.00													2	1							3
175.00										1		3	1		1					1	7
170.00					1		1	1	1	1		2	1	2	1	1	2				14
165.00								2	1	1	1	5	1	3	3						17
160.00						1		3	1	4	1	6	4	5	3		1	1			30
155.00							4	1	4	6	9	6	4	8	3	1	1	2			49
150.00				1		2		3	2	10	9	18	14	7	5		1				72
145.00			1				4	2	10	24	21	15	18	8	10	3	4	2			122
140.00			1		1	7	11	9	15	19	17	23	22	8	3	4	1		1		142
135.00				1	3	9	13	15	22	30	23	26	14	8	13	3		1			181
130.00		1		5	5	14	14	28	35	39	35	26	12	13	3			1			231
125.00			2	5	10	16	28	35	42	32	23	23	11	6	4	1	2	1			241
120.00				6	11	18	27	38	44	22	23	13	12	4	1		1	1			221
115.00		3	4	7	15	24	32	33	40	19	11	11	6	3	1						209
110.00		2	2	8	18	23	26	20	21	17	4	5	2	2	2						152
105.00		1	3	12	11	24	18	13	14	9	4	3	1								113
100.00	2		1	11	10	14	7	7	5	3		1									61
95.00		1	2	4	1	8	3	1	2												22
90.00			1		1		2														4
85.00					1																1
Totals	2	8	17	60	88	160	190	211	260	241	183	187	125	82	53	13	14	9	1	1	1905

Summary Statistics

	Mean	Std Dev	Regression Equations	SE-Est
Y-Weight	127.28	16.59	1.471X - 111.172	14.04
X-Stature	162.10	6.00	0.193Y + 137.536	5.08

C. A modest relationship: correlation coefficient = 0.533

LIP LENGTH AND MENTON-SUBNASALE LENGTH

MENTON-SUBNASALE LENGTH

LIP LENGTH	3 .95	4 .15	4 .35	4 .55	4 .75	4 .95	5 .15	5 .35	5 .55	5 .75	5 .95	6 .15	6 .35	6 .55	6 .75	6 .95	7 .15	7 .35	7 .55	TOTALS
5.75									2	1			1							4
5.55							1	2	2	3	2		1	1	2					14
5.35				1		1	4	2	4	2	3	5	1	1		1				25
5.15				2	1	1	15	12	9	11	10	8	3	1	1			1		75
4.95	1	1	1	3	10	8	13	17	16	21	19	13	9	1		3	1			137
4.75		1	2	7	9	17	23	35	27	32	29	22	9	7	2					222
4.55	1	2	4	7	16	26	38	53	53	45	44	26	18	6	5				1	345
4.35	2	3	3	8	11	22	41	41	57	30	37	21	7	2	5	4				294
4.15	1		6	8	16	37	47	69	66	48	44	25	6	9	6	2				390
3.95			5	6	10	19	28	30	46	18	26	10	6	4	3	2				213
3.75		1	1	2	2	12	16	19	26	12	19	5	3	2		1				121
3.55				1	2	3	6	6	9	6	11	2	1		1					48
3.35							1	1	4	2	2	1	1							12
3.15					1	1	1				1	1								5
Totals	5	8	22	45	78	147	234	287	321	231	247	139	66	34	25	13	1	1	1	1905

Summary Statistics

	Mean	Std Dev	Regression Equations	SE-Est
Y-Lip Length	4.38	0.42	0.058X + 4.057	0.42
X-Menton-Subnasale L	5.54	0.51	0.085Y + 5.169	0.51

D. A negligible, almost non-existent relationship:
correlation coefficient = 0.070

Figure 6. (continued)

In the first of these tables we may note that everybody with a specific value for stature has a common, or almost common, value for stature, maximum. In the fourth table, on the other hand, an individual's value for lip length provides virtually no indication of the size of her face length. In the other two tables, the patterns are intermediate between these two.

Two basic statistical concerns in this area of interrelationships are suggested by these tables. One is that of quantifying the differences in degrees of relationships so obvious here; this is the role played by the statistic known as the correlation coefficient. The second concern is that of establishing the pattern that values of one variable follow in relationship to a second; this is the role of the regression equation and the standard error of estimate. These two statistical concerns and the statistics involved are themselves well interrelated.

The correlation coefficient is the standard measure of the degree or intensity of the relationship between two variables. It ranges in value from 1.00, which indicates a perfect relationship, to 0.00, which indicates, on the other hand, no relationship. The first and fourth of our tables, with correlation coefficients of 0.998 and 0.128 come close to representing these extremes. The correlation coefficient can also fall in the range from 0.00 to -1.00 (this is somewhat rare for body size measurements) indicating that one variable tends to decrease in size as the other increases.

There are a substantial number of correlational measures. Of these, the most common for use with continuous data--such as our measurement data --is the Pearsonian product-moment correlation coefficient. Almost without exception this is referred to simply as the correlation coefficient. There are a variety of other types of correlation coefficients for use with categorized data (blood type, region of birth, etc.) but as these play little role in the solution of design problems we shall not discuss them.

Pearson's correlation coefficient derives from the related concept of the regression line or the regression formula. Given any two variables, we can set up an equation for estimating values for one variable in terms of the other. A typical example is the equation for estimating a man's sitting height from his stature shown in Figure 7. If the variables have a close relationship, the estimates given by the equation will be quite accurate. When, on the other hand, the degree of relationship is low or negligible, the estimates will have little accuracy.

No complete listings of the correlation coefficients for any of the sets of anthropometric data on which Volume II is based are included in this handbook. A few coefficients for USAF fliers and for Air Force Women are included, primarily for illustrative purposes, in Table 7. Correlation matrices for the USAF flying personnel surveys of 1967 and 1950, for Air Force Women (1968) and several other surveys are included in Churchill, Kikta, and Churchill (1977).

A. Calculations from Raw Data

X	Y	$X-\bar{X}$	$Y-\bar{Y}$	$(X-\bar{X})(Y-\bar{Y})$	$(X-\bar{X})^2$	$(Y-\bar{Y})^2$
7	6	0	1	0	0	1
9	7	2	2	4	4	4
5	1	-2	-4	8	4	16
4	3	-3	-2	6	9	4
10	8	3	3	9	9	9
Σ 35	25	0	0	27	26	34

a) The correlation coefficient: $r = \dfrac{\Sigma(X-\bar{X})(Y-\bar{Y})}{\sqrt{\Sigma(X-\bar{X})^2\Sigma(Y-\bar{Y})^2}} = \dfrac{27}{\sqrt{26\cdot 34}} = 0.91$

b) The regression line: $SD_x = \sqrt{\Sigma(X-\bar{X})^2/N} = \sqrt{26/5} = 2.28$; $SD_y = \sqrt{34/5} = 2.61$

i. to estimate y: $\alpha = r\ SD_y/SD_x = 0.91\cdot 2.61/2.28 = 1.04$

$\beta = \bar{Y} - \alpha\ \bar{X} = 5 - 1.04\cdot 7 = -2.28$

$SE_y = SD_y\sqrt{1-r^2} = 2.61\sqrt{1-(.91)^2} = 2.61\cdot .41 = 1.07$

$Y^* = 1.04\ X - 2.28$

ii. to estimate x: $\alpha = r\ SD_x/SD_y = 0.91\cdot 2.28/2.61 = 0.79$

$\beta = \bar{X} - \alpha\ \bar{X} = 7 - 0.79\cdot 5 = 3.05$

$SE_x = SD_x\sqrt{1-r^2} = 2.28\ \sqrt{1-(.91)^2} = 2.28\cdot .41 = 0.93$

$X^* = 0.79Y + 3.05$

B. Calculations Based on Computed Statistics

a) Simple regression equations:

i. to estimate sitting height from stature (USAF'67 data)

from Table VI: $r = 0.786$

from Volume II: sitting height - mean = 36.69", SD = 1.25"

stature - mean = 69.82", SD = 2.44"

$\alpha = r\ SD_y/SD_x = 0.786\cdot 1.25/2.44 = 0.403$

$\beta = \bar{Y} - \alpha\ \bar{X} = 36.69 - 0.403\cdot 69.82 = 8.55"$

$SE_y = SD_y\sqrt{1-r^2} = 1.25\ \sqrt{1-(.786)^2} = 1.25\cdot 0.618 = 0.77"$

$Y^* = 0.403\cdot X + 8.55$

For men 6 feet tall, we can estimate sitting height as

$Y^* = 0.403\cdot 72 + 8.55 = 37.57"$

two-thirds in a ±1 SD range: 37.57 - 0.77 = 36.8" to 37.57 + 0.77 = 38.3"

95% in a ±2 SD range: 37.57 - 1.54 = 36.0" to 37.57 + 1.54 = 39.1"

ii. to estimate stature from sitting height (the same data)

$\alpha = r\ SD_y/SD_x = 0.786\cdot 2.44/1.25 = 1.53$

$\beta = \bar{Y} - \alpha\ \bar{X} = 69.82" - 1.53\cdot 36.69 = 13.69$

$SE_y = SD_y\ \sqrt{1-r^2} = 2.44\ \sqrt{1-(.786)^2} = 2.44\cdot 0.618 = 1.51$

$X^* = 1.53\ Y + 13.69$

Figure 7. Correlation coefficients and regression equations: a few illustrative calculations.

For men with sitting heights of 34",

$Y^* = 1.53 \cdot 34 + 13.69 = 65.71"$

two-thirds in a ±1SD range: 65.71 - 1.51 = 64.2" to 65.71 + 1.51 = 67.2"

95% in a ±2SD range: 65.71 - 3.02 = 62.7" to 65.71 + 3.02 = 68.7"

b) Multiple correlation and regression:

i. correlation of X_3 with the combination of X_1 and X_2:

$$R = \sqrt{\frac{r_{1,3}^2 + r_{1,2}^2 - 2r_{1,2} \cdot r_{1,3} \cdot r_{2,3}}{1 - r_{1,2}^2}}$$

to estimate chest circumference (X_3) in terms of stature (X_1) and weight (X_2): (USAF'67 data)

$r_{1,3}$ = correlation of stature with chest circumference = 0.257

$r_{2,3}$ = correlation of weight with chest circumference = 0.799

$r_{1,2}$ = correlation of stature with weight = 0.533

$$R = \sqrt{\frac{(.257)^2 + (.799)^2 - 2(.533)(.257)(.799)}{1 - (.533)^2}} = \sqrt{\frac{0.486}{0.716}} = 0.824$$

ii. to estimate X_3 from X_1 and X_2

$$\frac{X_3^* - \bar{X}_3}{SD_3} = \beta_1 \frac{X_1 - \bar{X}_1}{SD_1} + \beta_2 \frac{X_2 - \bar{X}_2}{SD_2}$$

where $\beta_1 = \frac{r_{1,3} - r_{1,2} \cdot r_{2,3}}{1 - r_{1,2}^2} = \frac{.257 - .533 \cdot .799}{1 - (.533)^2} = -0.236$

$$\beta_2 = \frac{r_{2,3} - r_{1,2} \cdot r_{1,3}}{1 - r_{1,2}^2} = \frac{.799 - .533 \cdot .257}{1 - (.533)^2} = 0.925$$

The standard error of estimate = $SD_3 \sqrt{1 - R^2} = 0.567\ SD_3$

$$X_3^* = \beta_1 \frac{SD_3}{SD_1} X_1 + \beta_2 \frac{SD_3}{SD_2} X_2 + \bar{X}_3 - \beta_1 \frac{SD_3}{SD_1} \bar{X}_1 - \beta_2 \frac{SD_3}{SD_2} \bar{X}_2$$

Since, $\bar{X}_1$ = 69.82", SD_1 = 2.44"

$\bar{X}_2$ = 173.6 lb, SD_2 = 21.4 lb

$\bar{X}_3$ = 38.80", SD_3 = 2.50"

$$X_3^* = -.236 \frac{2.50}{2.44} (X_1 - 69.82) + .925 \frac{2.50}{21.4} (X_2 - 173.6) + 38.80$$

$$= -.242\ X_1 + .108\ X_2 + 36.95$$

Thus, our estimate of chest circumference of a man 6' tall who weighs 200 pounds is

$$X_3^* = -.242 \cdot 72 + .108 \cdot 200 + 36.95 = 41.13"$$

Since $SE_y = 2.50 \cdot \sqrt{1-(.824)^2} = 1.42"$, we can expect that about two-thirds of such men will have chest circumferences in the range $X_3^* \pm SE_y$:

41.13 ± 1.42 = 39.7" to 42.6", and 95% in the range $X_3^* \pm 2SE_y$ =

41.13 ± 2.84 = 38.3" to 44.0"

Figure 7. (continued)

TABLE 7
SELECTED CORRELATION COEFFICIENTS FOR USAF FLIERS AND AIR FORCE WOMEN*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1. Age		.223	.048	-.023	.039	-.055	.091	-.072	.233	.287	.234	.219	.149	.146	.194	.095	.118	.190	.189	.089
2. Weight	.113		.533	.457	.497	.431	.481	.370	.835	.799	.824	.886	.495	.768	.770	.403	.304	.290	.264	.358
3. Stature	-.028	.515		.927	.914	.849	.801	.728	.334	.257	.279	.360	.456	.329	.348	.331	.318	.136	.267	.199
4. Chest height	-.028	.483	.949		.897	.862	.673	.731	.271	.183	.216	.289	.412	.266	.276	.284	.284	.085	.222	.162
5. Waist height	-.033	.422	.923	.930		.909	.607	.762	.308	.238	.238	.336	.409	.293	.318	.306	.297	.123	.225	.200
6. Crotch height	-.093	.359	.856	.866	.905		.467	.788	.264	.190	.221	.246	.380	.277	.225	.294	.280	.089	.205	.172
7. Sitting height	-.054	.457	.786	.681	.580	.453		.398	.312	.239	.236	.383	.384	.277	.379	.294	.275	.136	.248	.146
8. Popliteal height	-.102	.299	.841	.843	.883	.880	.485		.230	.172	.186	.201	.327	.249	.181	.235	.253	.087	.185	.189
9. Shoulder circumference	.091	.831	.318	.300	.261	.212	.291	.182		.810	.775	.717	.581	.719	.606	.330	.248	.252	.217	.313
10. Chest/bust circumference	.259	.832	.240	.245	.203	.147	.171	.114	.822		.796	.674	.370	.706	.551	.273	.204	.255	.176	.273
11. Waist circumference	.262	.856	.224	.212	.142	.132	.167	.068	.720	.804		.722	.382	.886	.600	.281	.149	.267	.174	.310
12. Buttock circumference**	.105	.922	.362	.334	.278	.217	.347	.149	.744	.766	.852		.396	.668	.893	.310	.214	.238	.180	.269
13. Biacromial breadth	.003	.452	.378	.335	.339	.282	.349	.316	.555	.401	.288	.355		.401	.361	.311	.239	.178	.266	.211
14. Waist breadth	.214	.852	.287	.260	.215	.195	.216	.133	.715	.801	.936	.849	.327		.576	.292	.168	.263	.182	.296
15. Hip breadth	.105	.809	.414	.380	.342	.283	.376	.221	.632	.647	.724	.895	.340	.760		.265	.183	.188	.155	.215
16. Head circumference	.110	.412	.294	.251	.233	.188	.287	.194	.327	.340	.309	.330	.251	.310	.288		.692	.430	.273	.299
17. Head length	.054	.261	.249	.218	.208	.170	.244	.175	.201	.196	.158	.195	.179	.164	.166	.779		.115	.311	.113
18. Head breadth	.122	.305	.133	.097	.089	.066	.132	.075	.245	.271	.265	.252	.188	.268	.227	.521	.058		.174	.497
19. Face length	.119	.228	.275	.220	.226	.199	.253	.193	.162	.172	.129	.186	.187	.151	.161	.315	.289	.148		.144
20. Face breadth	.233	.453	.190	.169	.142	.099	.185	.098	.401	.421	.412	.394	.278	.410	.364	.464	.131	.660	.206	

*Air Force Women '68 above diagonal; USAF Fliers '67 below diagonal.

**Hip circumference 9" below waist for Air Force Women '68.

Logically, there are two correlation coefficients for each pair of variables: the one defined in terms of how well we can estimate Y from X and that defined in terms of how well we can estimate X from Y. Fortunately these two are numerically equal and need not be distinguished. This is not true when regression equations corresponding to curved lines are used or when more than one variable is used in the estimating process. There are, of course, different regression lines for each variable.

The basic definition for the correlation coefficient for X and Y can be written as follows:

$$r = \frac{\Sigma(X-\overline{X})\ (Y-\overline{Y})}{\sqrt{\Sigma(X-\overline{X})^2 \cdot \Sigma(Y-\overline{Y})^2}}$$

and is illustrated in Figure 7. We can argue that this formula is at least a reasonable one as a measure of relationship. The terms in the denominator are always positive, but the terms in the numerator can be either positive or negative. They will be positive when X and Y are both above average and when both are below average; they will be negative whenever X is above average and Y below average or vice versa. Since terms of one sign cancel those of the other sign, the size of the numerator (and therefore of the correlation coefficient) will reflect the extent to which terms of one sign predominate. We have used the letter r here to designate the correlation coefficient; this is standard practice. When it is necessary to specify the relevant variables, we may write $r_{1,2}$ or $r_{x,y}$ or some similar expression.

There is a bit more to this formula than noting how often individuals are, on the one hand, either below or above the mean on both of a pair of measurements and how often, on the other hand, they are above the mean on one and below the mean on the other. Still, this concept of the correlation coefficient is accurate enough to provide a useful basis for judging the size of a correlation coefficient. By replacing the mean with the median in this concept (which will make little difference for most body size measurements) we can reduce our data for a pair of variables to a simple 2x2 table:

Measurement Y	Measurement X: Below Median	Measurement X: Above Median
Above Median	B	A
Below Median	A	B

and take as an approximation:

$$r(\text{approx}) = \frac{A - B}{A + B}$$

Thus, if in a group of 200 pilots, 75 of the 100 men who are above the median value for weight are also above the median in stature and vice versa, we would have the table:

	-	+
+	25	75
-	75	25

A-B = 75-25 = 50

A+B = 75+25 = 100

r (approx) = 50/100 = 0.5

Restated, this formula suggests that out of every 100 individuals who are above the median on one measurement, the number who will also be above the median on a second measurement is about:

50 whenever r = 0.0
55 whenever r = 0.1
60 whenever r = 0.2
65 whenever r = 0.3
70 whenever r = 0.4
75 whenever r = 0.5
80 whenever r = 0.6
85 whenever r = 0.7
90 whenever r = 0.8
95 whenever r = 0.9

This relationship is an approximate one but is reasonably good for the purpose of evaluating the degree of relationship that a correlation coefficient, based on body size data, represents.

Another quite important interpretation of the correlation coefficient is in terms of the accuracy of the regression equation estimates. It is customary to measure this accuracy by a statistic--the standard error of estimate--which is similar to the standard deviation but is based on the differences between the actual data values and the estimated values, rather than on the differences between the data and the arithmetic mean. The standard error of estimate is defined as

$$SE_y = \sqrt{\Sigma(Y-Y^*)^2/N}$$

where Y* represents the regression estimates and Y the actual values. By algebraically manipulating this formula and the one for the correlation coefficient we arrive at the important relationship between these two statistics:

$$SE_y = SD\sqrt{1-r^2}$$

Note that, as we should expect, SE is zero for perfect correlations (r = +1 or -1) and equals the standard deviation where r = 0. We may further observe that, since the correlation coefficient appears here as a squared value, a negative value of r has the same effect as a positive one of equal magnitude.

Just as, in general, two-thirds of a set of data lie within a standard deviation of the mean, so too about two-thirds of a set of estimates will lie within one standard error of estimate of the actual values. Similarly, about 95% of the data fall within two standard deviations of the mean, and about 95% of the estimates fall within two standard errors of estimate of the actual values. Reversing these last statements, we find that about two-thirds of the actual values lie within a band running from a standard error of estimate above the regression line to a standard error of estimate below it, 95% lie within the ± 2 SE band, and so forth. Thus, referring to Figure 8, our best estimate of the sleeve inseam of a USAF flyer who is 180 centimeters tall is about 49.3 centimeters, the regression value, and the chances are about two out of three that the inseam measurement is somewhere between 47.5 and 51.1 centimeters since $SE_y = 1.8$ cm.

The standard error of estimate, like the regression value, has a second important identity: the standard error is both a measure of the accuracy of a single estimate and, at the same time, the standard deviation for Y of all individuals with a fixed X-value. The regression value is both our best estimate of Y for an individual with a specified value of X and the mean value of Y for these individuals. Thus, we can say both:

a. for an individual with a stature of 180 centimeters, our best estimate of his sleeve inseam is 49.3 cm and there are two chances in three that this estimate will be in error by no more than 1.8 cm; and
b. for the group of men with statures of 180 centimeters, the mean sleeve inseam is 49.3 cm and the standard deviation is 1.8 cm.

The relationship between the standard error of estimate and the correlation coefficient is further illustrated by the following:

r	SE_y	r	SE_y	r	SE_y
0.00	100% SD	0.40	91.7% SD	0.80	60.0% SD
0.10	99.5% SD	0.50	86.6% SD	0.90	43.6% SD
0.20	98.0% SD	0.60	80.0% SD	0.95	31.2% SD
0.30	95.4% SD	0.70	71.4% SD	0.99	14.1% SD

Regression Equations

The regression equation has already been more or less defined as the equation or formula for estimating one variable's value from that of a related variable. Tacitly we have assumed that this equation was linear in nature; that is, that its graph is a straight line, and that our equation is the "best" possible. These are universal assumptions when working with anthropometric data.

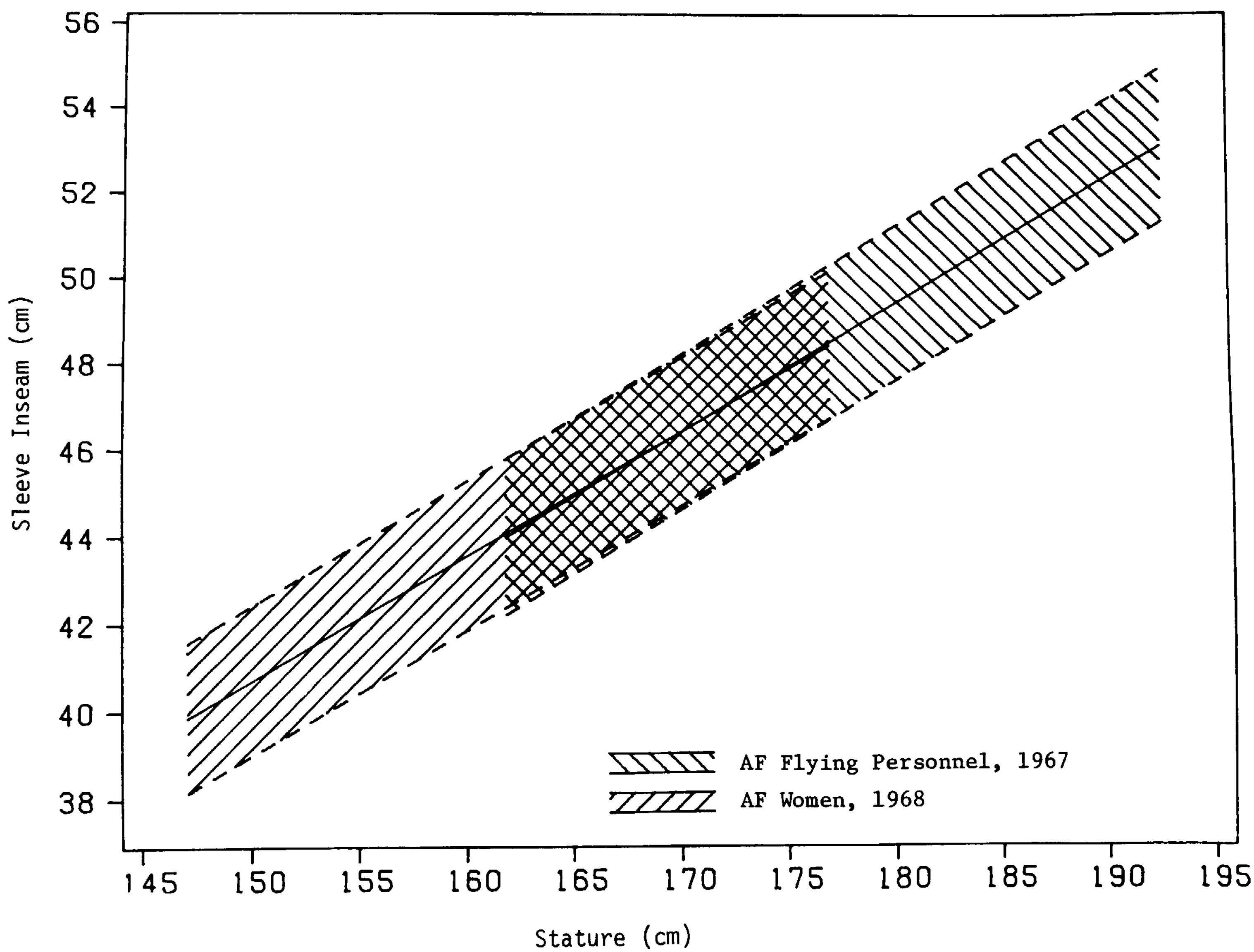

Figure 8. Regression bands: regression values ± 1 SE_y.

A useful way of writing the formula for the regression equation for Y in terms of X is this:

$$\frac{Y^*-\overline{Y}}{SD_y} = r\,\frac{X-\overline{X}}{SD_x}$$

The equation, written in this form, points up the statement about the estimates "regressing" to the means. If X is "K" standard deviations from its mean, the estimate Y* will be K•r standard deviations from its mean. Since r cannot exceed unity, and in any practical situation never equals it, K • r will always be less--in absolute value--than K and in standard deviation units, Y* will be closer to $\overline{Y}$, than X is to $\overline{X}$. In our earlier statement that the regression estimate of the weight of a man who was 2SD above the mean in stature would be about 1SD above mean weight, we used 0.5 as the approximate correlation between stature and weight, a fairly good estimate of the correlation found for many series of data obtained by measuring healthy, youngish adults.

A more conventional form of the regression equation--one absolutely algebraically equivalent to the one just given--is:

$$Y^* = \alpha X + \beta \quad \text{where } \alpha = rSD_y/SD_x;\ \beta = \overline{Y} - \alpha\,\overline{X}$$

The value of α , the coefficient of X, is the slope of the regression line; β is, in theory, the Y-intercept of the line, that is, the value of Y* for X=0. We have qualified this last phrase with "in theory" because the range of values for which we may reasonably assume the regression line to be valid does not exceed the range of the data on which it is based. USAF flying personnel measured in 1967 ranged in stature from about 62" to 77"; no attempt should be made to use regression equations based on the data from these men with values of stature outside this range. In addition, one should expect regression estimates to be less accurate when based on values near the ends of the range than those based on values close to average.

The computation of regression equations, based on this last formula, is illustrated in Figure 7. Needless to say, the calculations based on a sample of five are intended to illustrate a formula and not to suggest that it is appropriate to use correlational techniques with very small samples.

Intercorrelations of Body Size Data--High or Low?

How big correlation coefficients for anthropometric variables tend to be is a question without a precise answer. While the correlational coefficients obtained from a particular set of data will depend somewhat on the individuals measured, they will depend even more on the measurements included in the data. The "typical" coefficient for a survey in which only a few major dimensions were measured will, almost certainly, be much higher than the "typical" value for a major survey in which a large number of major and minor dimensions were measured.

One of the most comprehensive analyses of a large batch of anthropometric correlations appears in Anthropometry of Air Force Women (Clauser et al. 1972). It may be of interest to consider the results of this analysis since it is reasonable to assume that these results are, in broad terms, about the same as those we would find by studying data from other large surveys.

The distribution of the 7,626 correlation coefficients based on age and the 123 body size measurements made on the entire sample in the Air Force Women's survey is summarized in Table 8 and Figure 9. Several things are clear from Figure 9. The size of the correlation coefficients ranges from rather small, negative values almost to a perfect correlation of 1.00. Most of the values are positive; if we ignore the values which are not significantly different from zero (the shaded area in Figure 9), there are almost no negative values. Despite this wide range, most of the correlation coefficients lie between 0.1 and 0.4, values which may sometimes be of interest but which are of almost no significance in design problems. The most common (model) correlation coefficient is equal to a little more than 0.2, corresponding to a rather trivial level of intercorrelation.

To explore the question of how the correlation coefficients are distributed when the variables involved are of a particular type, the 124 variables involved in this analysis were divided into 9 categories: (1) age, (2) weight; (3) skinfold measurements; (4) heights (excluding lateral malleolus height), reaches, and long bone measurements; (5) torso breadths and depths; (6) torso (including neck) circumferences and horizontal surface measurements; (7) limb breadths and circumferences; (8) hand and foot measurements (including lateral malleolus height); and (9) head and face measurements.

Table 8 shows the distributions obtained when the variables are divided into these categories. Section I of this table presents essentially the same information as is contained in Figure 9: the range of the correlation coefficients is from a minimum of -0.21 to a maximum 1.00* with a median of 0.24. Section II summarizes the patterns, by category, for the correlations of all the variables with the variables in each of the nine categories. Only the correlations with weight show a pattern of values distinctly higher than the pattern for the total distribution. The median value for the correlations with weight is 0.50, but for none of the eight other categories were as many as 25% of the correlations that large.

Section III carries the process of breaking down the distribution one step further and considers, at each step, only those correlations involving variables from a specified pair of categories. Of the 37 sets of coeffi-

*Lest this value be regarded as a refutation of the statement made several times in this chapter that the correlation coefficient is never +1.00 in any realistic situation, we note that this value is really 0.998. As it is the correlation between stature measured two ways, its large size is not surprising.

TABLE 8

DISTRIBUTION OF CORRELATION COEFFICIENTS BY VARIABLES, GROUPS OF VARIABLES, AND ENTIRE GROUP

(from Anthropometry of Air Force Women by Clauser et al., 1972)

I. Total Series Summary

	Percentiles											
	MIN	1	5	10	25	50	75	90	95	99	MAX	N
	-.21	-.02	.05	.08	.15	.24	.39	.62	.73	.88	1.00	7626

II. Major Groups Summaries

Group	MIN	1	5	10	25	50	75	90	95	99	MAX	N
1. Age	-.08		-.02	.00	.05	.12	.19	.28	.29		0.33	123
2. Weight	0.08		.17	.22	.31	.50	.74	.80	.82		0.90	123
3. Skinfolds	-.10	-.07	-.02	.00	.04	.12	.36	.56	.61	.68	0.72	486
4. Heights	-.21	-.05	-.04	.09	.16	.25	.36	.58	.72	.90	1.00	3531
5. Breadths	-.08	.04	.08	.12	.19	.29	.49	.66	.72	.84	0.89	1298
6. Circumferences	-.10	.03	.08	.11	.19	.30	.47	.65	.73	.84	0.94	2166
7. Limb C's & B's	-.06	.03	.08	.12	.19	.31	.45	.64	.72	.81	0.98	2270
8. Hand & Foot	-.07	.00	.06	.10	.18	.26	.35	.46	.57	.68	0.74	723
9. Head & Face	-.14	-.02	.03	.06	.11	.16	.23	.28	.31	.62	0.95	3161

III. Cross Group Summaries

	MIN	1	5	10	25	50	75	90	95	99	MAX	N
1 & 4					.00	.05	.08					33
1 & 5						.23						11
1 & 6						.23						19
1 & 7						.15						20
1 & 9					.07	.09	.14					29
2 & 4					.41	.46	.53					33
2 & 5						.77						11
2 & 6						.79						19
2 & 7						.78						20
2 & 9					.19	.26	.30					29
3 & 4	-.08		-.03	-.01	.02	.05	.10	.18	.22		0.29	132
3 & 5					.36	.47	.57					44
3 & 6				.16	.26	.37	.54	.61				76
3 & 7				.20	.28	.41	.54	.63				80
3 & 8					.04	.05	.08					24
3 & 9	-.10		-.03	-.02	.01	.05	.10	.16	.21		0.27	116
4 & 4	-.21	-.09	.16	.25	.39	.63	.75	.87	.91	.97	1.00	528
4 & 5	-.08		.08	.13	.18	.24	.31	.38	.42		0.59	363
4 & 6	-.10	.06	.11	.15	.20	.27	.35	.44	.53	.70	0.82	627
4 & 7	-.06	.01	.08	.11	.18	.29	.35	.42	.45	.49	0.59	660
4 & 8	0.01		.16	.21	.29	.34	.46	.60	.66		0.70	198
4 & 9	-.13	-.05	.03	.07	.12	.17	.22	.27	.29	.33	0.36	957
5 & 5				.31	.50	.59	.67	.71				55
5 & 6	0.09		.26	.33	.44	.56	.69	.76	.83		0.89	209
5 & 7	0.25		.29	.31	.37	.50	.64	.70	.72		0.84	220
5 & 8				.11	.19	.23	.30	.35				66
5 & 9	-.02		.05	.07	.11	.17	.22	.26	.29		0.35	319
6 & 6	0.07		.21	.29	.44	.54	.66	.79	.83		0.94	171
6 & 7	0.11		.23	.29	.37	.48	.62	.73	.76		0.89	380
6 & 8	0.08		.12	.13	.19	.26	.32	.36	.40		0.49	114
6 & 9	-.03	.00	.04	.07	.10	.16	.22	.27	.30	.33	0.37	531
7 & 7	0.33		.37	.39	.43	.54	.69	.80	.92		0.98	190
7 & 8	0.07		.11	.17	.23	.32	.39	.49	.55		0.71	120
7 & 9	0.00	.03	.06	.08	.12	.17	.23	.26	.29	.32	0.36	580
8 & 8						.47						15
8 & 9	-.04		.05	.08	.13	.19	.23	.28	.31		0.40	174
9 & 9	-.14		.01	.04	.10	.16	.30	.53	.78		0.95	406

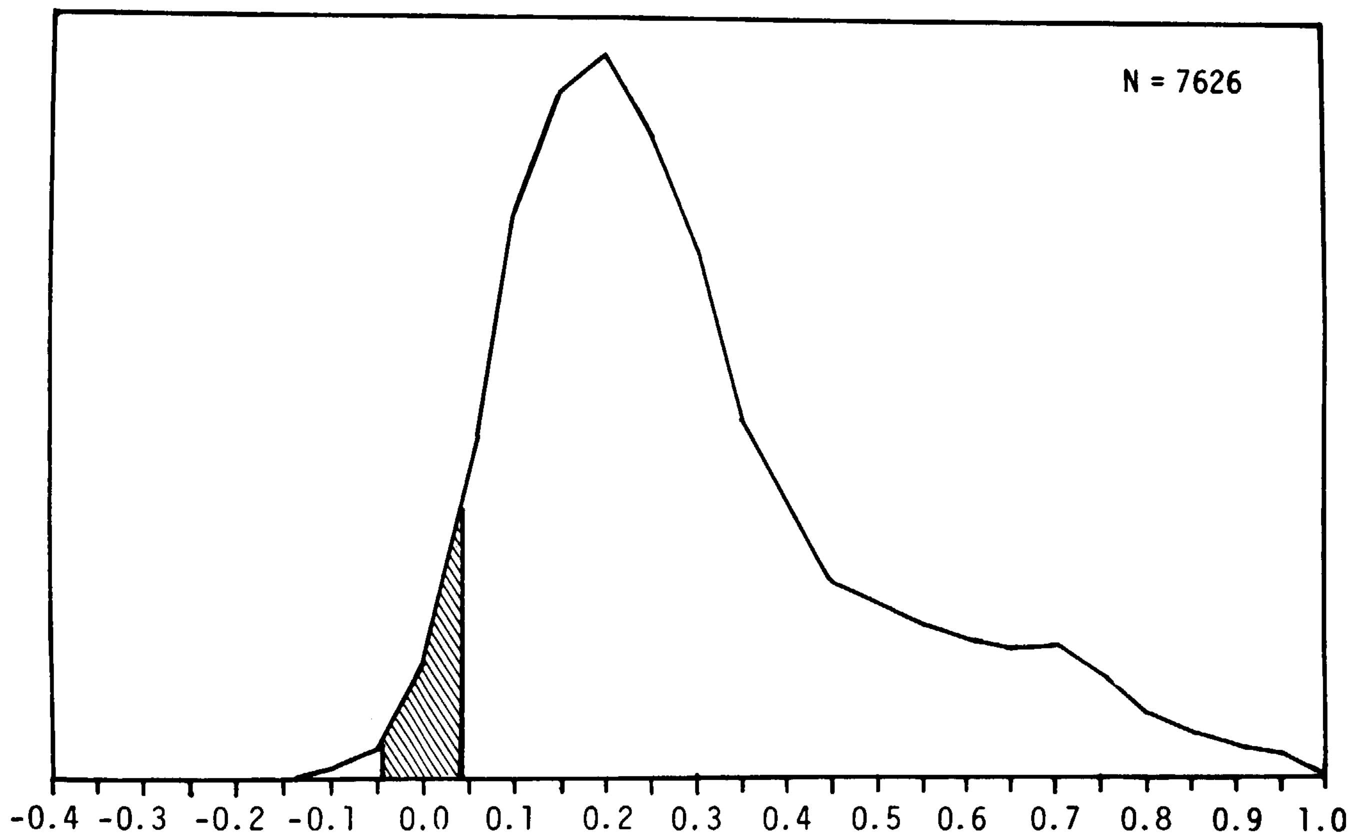

Figure 9. Distribution of correlation coefficients (from Clauser et al. 1968).

cients with more than 10 values, only the following nine had median values of at least 0.50:

weight and breadths-depths	median r = 0.77
weight and circumferences	median r = 0.79
weight and limb circumferences-breadths	median r = 0.78
heights and heights	median r = 0.63
breadths-depths and breadths-depths	median r = 0.59
breadths-depths and circumferences	median r = 0.56
breadths-depths and limb circumferences-breadths	median r = 0.50
circumferences and circumferences	median r = 0.54
limb circumferences-breadths and limb circumferences-breadths	median r = 0.54

Five other groups almost reached the 0.50 level: weight and heights (0.46), skinfolds and breadths-depths (0.47), skinfolds and circumferences (0.47), circumferences and limb circumferences-breadths (0.48), and hand-foot measurements and hand-foot measurements (0.47).

This breakdown has demonstrated that there are a few categories of body size measurements for which the correlation coefficients are, typically, of at least modest size. It also demonstrates that the overall distribution of correlations is weighted heavily towards the low end of the scale by large numbers of correlations which would rarely, if ever, be of any importance. Over a third of the correlation coefficients, for example, are correlations between measurements of the head and face and measurements of other parts of the body. That these 2,755 correlations have a median value of 0.16 is probably of no importance in any real design problem. On the other hand, the fact that the correlation coefficients for one head-face measurement with another also have a median value of 0.16 presents serious problems in the design of helmets and masks.

Interrelationships--More than Two Variables at a Time

The regression equation concept--that of estimating values of one variable from values of a second--is easily extended to the concept of the multiple regression equation. Using such an equation, we can estimate a man's weight from his height and his chest circumference, or from his height and his head, shoulder, chest, waist, and buttock circumferences, or from any other combination of two, three, four, or more variables. The quality of these estimates, as measured by their agreement with actual values, can be expressed in terms of the multiple correlation coefficient and the multiple standard error of estimate, statistics absolutely equivalent* to the simple correlation coefficient and the simple standard error of estimate.

*The multiple correlation coefficient is always considered to be positive.

All these multivariate statistics can be computed directly from the simple correlation coefficients and the means and standard deviations; we shall limit our display of formulas here to the inclusion in Figure 7 of the formula for the correlation between one variable and a pair of other variables. Other formulas are included in Churchill et al. (1977); many examples of multiple regression equations for body size measurements are given in the report of the Air Force Women's survey.

When we use a multiple regression equation based on a pair of variables, our input into the equation contains more information than when we use a simple equation based on either member of the pair. With more information we should get more accurate estimates and, as a matter of fact, we do. The multiple correlation of, say Z with X and Y will always be larger than both the simple correlations of Z with X and Z with Y. Unfortunately, the multiple correlation will often be only trivially larger than the larger of the two simple correlations. This is all too true with body size correlations. It is commonly assumed, for example, that the multiple regression equations based on stature and weight provide good estimates of most anthropometric measurements. While this is, in large measure, true, it is also true that most of the time these estimates are not much better than those obtained from the better of two simple equations. In the 1968 Air Force Women's data, for example, for 121 measurements, the multiple correlation coefficient based on stature and weight provided an improvement over the simple equations (based on either stature or weight) of no more than 0.01 for about half the measurement and an improvement of from 0.01 to 0.02 for 27 measurements. For only 35 of the measurements did the increase exceed 0.02. A typical case was that for thumb-tip reach which correlated 0.433 with weight and 0.646 with stature; the multiple correlation with weight and stature, 0.655, represented only a minor increase.

A relatively new approach to multiple regression is worth mentioning briefly--"stepwise" regression equations. This is a technique which became practical only with the advent of the modern computer. A matrix of correlation coefficients is entered into the computer which then computes for each variable the best equation based on a single other variable, then the best equation based on two variables, and so on for as many equations as are desired. The results obtained by applying this approach to a set of survey data are often interesting. Applied blindly, however, this technique is not likely to satisfy the hope of those who expect it to identify a small group of variables on which to base equations for estimating all the other variables. The resulting equations are, unfortunately, all too likely to use a substantial portion, if not almost all, of the variables. The 121 one-predictor equations for the 1968 Air Force Women's data used well over half that many predictors, the two-predictor equations used about 100 different variables, and the three-predictor ones all but half a dozen of the variables. About a dozen two-variable combinations were "best" for predicting two variables apiece, but none was best for more than two. Nonetheless, this approach would seem to have potential usefulness in the analysis of body size interrelationships, and references to it appear from time to time in anthropometric literature.

A Mathematical Model for Body Size Data

There are at least two fairly common ways of determining the circumference of a bicycle wheel: first, we can measure it directly; secondly, we can measure the distance from the center of the axle to the edge of the wheel and multiply the result by 2π. When we use this second method, we use the circle as a mathematical model for the wheel and make use of the fact that, for this model, $C=2\pi R$.

In working with anthropometric data, we often have a similar choice of procedures. We can measure some statistical value directly or we can estimate it indirectly using a mathematical model appropriate to such data. For most body size data, the most appropriate model is the normal distribution, the "normal curve," (see, for example, Figure 2). Just as there are circles of many sizes, so too there are an infinite number of normal distributions corresponding to all possible values of the mean and the standard deviation.*

No set of data fits this model perfectly, but then, there has never been a perfectly round bicycle wheel. Sometimes neither model may be adequately close to the real thing: subscapular skinfold measurements and wheels with flat tires are, perhaps, somewhat equivalent examples of this. Most body size measurements, on the other hand, fit our mathematical model within usual design tolerances and over the usual range of design values; both these reservations are real--and also realistic. The proportion of USAF pilots between $\overline{X} \pm 1SD$ values in stature is, according to the tables of the normal distribution, 68.268%; in more reasonable and more realistic words, we can expect about two-thirds of the pilots to fall in this range. For most designs, the two-thirds is likely to be adequate and accurate; the use of 68.268%, in contrast, will probably make as much sense as using 3.14159265 for π in determining the circumference of the wheel.

Virtually all men have statures within 3 or 3.5 standard deviations of the mean value; only an occasional individual will fall outside this range and what his stature will be and, relatively speaking, how many such individuals exist in any group of men, are matters too erratic for close

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*The mathematical statement of the normal distribution is that, in a population of values with a mean of M and a standard deviation of SD, the proportion of values less than any value X_0 is given by the integral:

$$P(X \leq X_o) = \frac{1}{SD\sqrt{2\pi}} \int_{-\infty}^{X_o} e^{-\frac{1}{2}\left(\frac{t-M}{SD}\right)^2} dt$$

where $\Pi = 3.14....$ and $e = 2.78....$ have their usual meanings. The value of the integral cannot be expressed as a simple function of X_o but tables of the integral are legion. This integral is closely related to, but not quite the same as, the error function (ERF) sometimes used in engineering studies.

prediction. The number of men in a group the size of the USAF flying personnel sample (N=2,420) more than 4 standard deviations above the mean in stature (or any other measurement) is, on the basis of the normal distribution, just about one. In practice, we're likely to find one, or two, or three men this tall--or, quite often, none at all. No matter what the actual number is, our mathematical model has indicated, quite accurately, that there are so few men in this "tail" of the distribution that only a most unusual design plan need concern itself with them.

The importance of a mathematical model is not only its usefulness in providing an alternative to direct computation for determining statistical values, but also in the help it provides in generalizing about statistical problems and in developing approaches to their solutions. The mathematical model at times also makes it possible to determine statistical values which cannot be determined directly or to provide such values in a form not possible by direct computation. This is true, of course, not only of the normal distribution model for body size (and many other types of data) but for other models (binomial distribution, Poisson distribution, hypergeometric distribution, etc.) more appropriate to other types of statistical data.

Percentiles and Related Values

One of the most important applications of the normal distribution model has already been mentioned several times in this chapter--the estimation of percentiles and similar values in terms of the mean and standard deviations, and the estimation of the proportions of a set of data which lie within specified ranges. Tables 2, 5 and 6 are based directly on the evaluation of the integral of the normal distribution.

The Bivariate and Multivariate Models

The mathematical model of the normal distribution for a single anthropometric variable can be extended easily--and with reservations similar to those already noted--to the joint distribution of two or more body size measurements. While the model for a single variable is determined by matching model and actuality in terms of a mean and standard deviation, in the two-variable case, the matching is done in terms of five statistics: the two means, the two standard deviations, and the correlation coefficient. The model can be extended to any number of variables, depending in each case on the means and standard deviations of the variables involved plus all their correlation coefficients. The mathematics involved in using this model becomes somewhat complex as the number of variables increases, but computer programs are available for a variety of uses of the two- and three-variable forms and for some uses involving essentially any number of variables.*

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*Formulas for, and related to, these models are included in Churchill et al. (1977). Relevant computer programs will be included in Computer Programs

We shall discuss three uses of the bivariate normal distribution model: the construction of equal probability ellipses, the construction of artificial bivariate tables, and the determination of proportions disaccommodated by two-variable designs.

Equal Probability Ellipses

The concept of the equal probability ellipse can be approached by considering a design range defined by a pair of complementary percentiles, say, the 5th and the 95th. The range of values between these percentiles has two important characteristics:

(1) a specified proportion (i.e., 90%) of the data lies in this range, and
(2) every value within the range has a higher probability than every value outside it.

Combined, these two characteristics add up to the fact that the 5th-95th percentile range is the shortest range containing 90% of the data.

The extension of these concepts to the two-variable case leads to the notion of the equal probability ellipse, as shown in Figure 10. These ellipses have been constructed so that, like the 5th-95th percentile range:

(1) 90% of the pairs of values lie inside the ellipse,
(2) every point inside the ellipse corresponds to a higher probability (or relative frequency) than every point outside the ellipse, and, consequently,
(3) the interior of the ellipse is the smallest region (as measured in SD units) containing 90% of the data.

It is not particularly clear how one would use an ellipse in establishing design limits for a piece of equipment or clothing. Nonetheless, these ellipses are useful in indicating the major distribution of the data for a pair of variables. When data for two or more groups of individuals must be considered in a single design program, the appropriate ellipses for the several groups, drawn on a single graph, may help to indicate the nature of the problems involved; Figures 10 and 11, for example, illustrate well how differently different types of anthropometric measurements for women relate to the same measurements for men.

It may be worth noting in passing how a 90% constant-probability ellipse differs from the rectangle whose sides correspond to the 5th and 95th percentiles for the two variables involved. First, the relative frequency of the individuals who fall in such a rectangle, i.e., the individuals who are within the 5th-95th percentile range on both variables, is

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for Anthropometric and Statistically Similar Data by Churchill, Kikta, and Churchill (in preparation) and can be obtained from Webb Associates, Box 308, Yellow Springs, Ohio 45387.

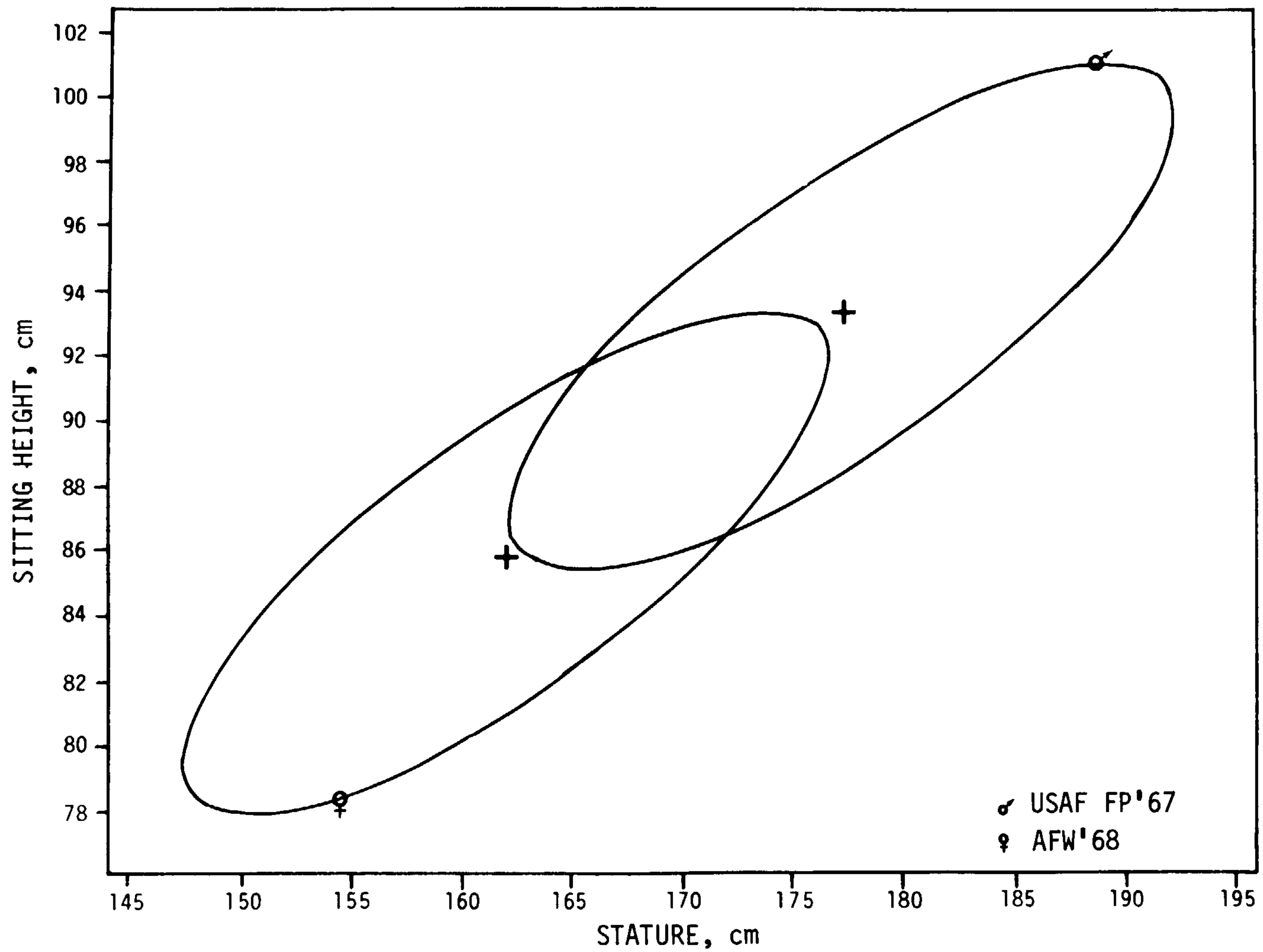

Figure 10. Ninety-five percent probability ellipse for sitting height and stature.

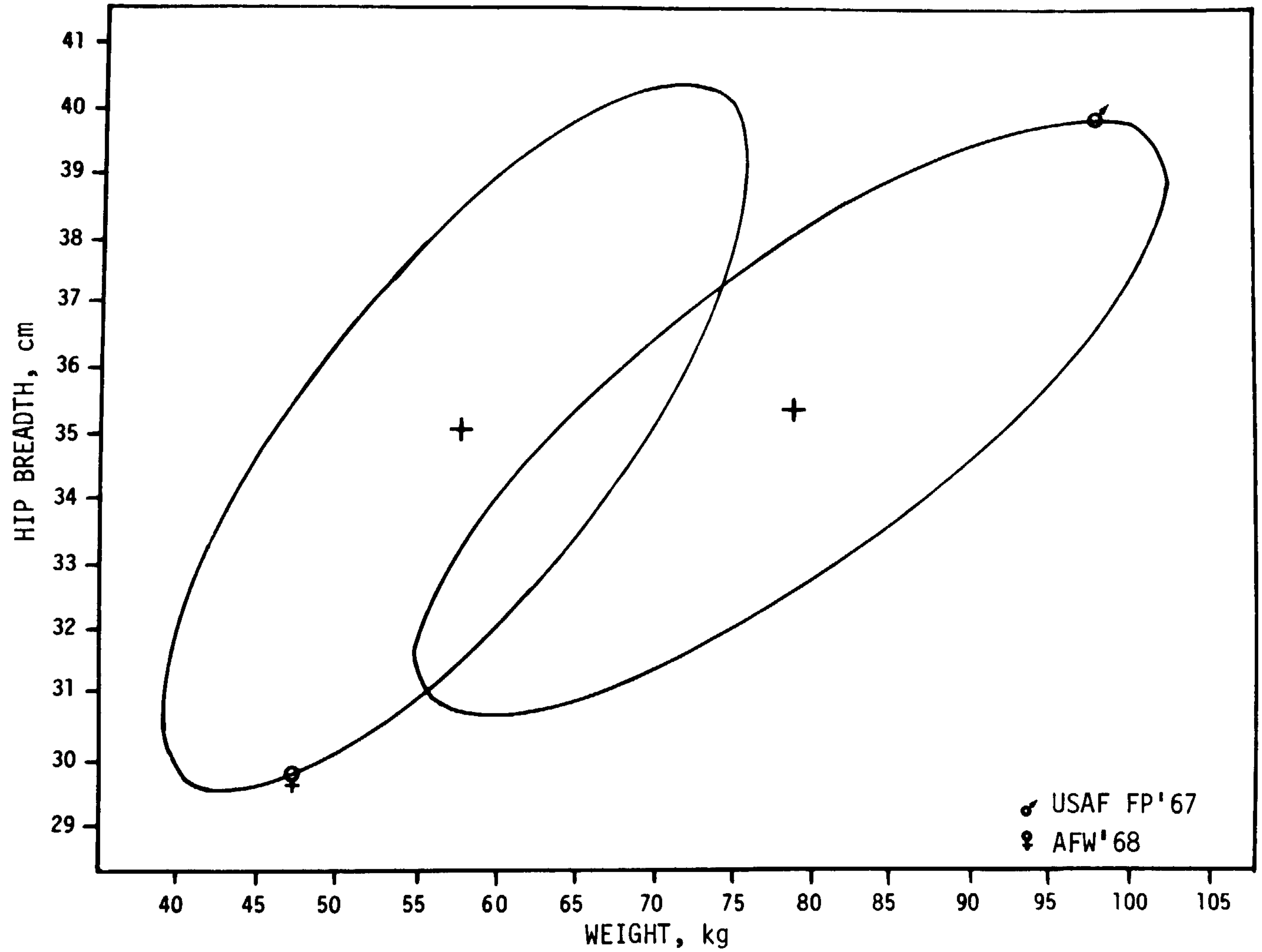

Figure 11. Ninety-five percent probability ellipse for weight and hip breadth.

almost certainly not 90%. For variables with almost zero correlations--face length and face breadth, for example--the rectangle will contain only about 81% of the data. The higher the correlation--its sign is irrelevant--the higher the percentage within the rectangle, with 90% corresponding only to a perfect correlation. Second, the rectangle will include, in its lower right and upper left corners, if the variables are positively correlated, some relatively atypical individuals--the 6'1" pilot who weighs 140 pounds and the 5'6" one who weighs 210 pounds, for example--while excluding much more likely individuals, such as a 6'1" pilot who weighs 215 pounds. This problem probably cannot be avoided if rectangular design limits must be used. The question of how to obtain rectangular design limits which include a specified proportion of the data will be addressed below.

Artificial Bivariate Tables

A bivariate table, like those pictured in Figure 6 is made by recording the number of individuals whose values for a pair of variables fall within specified limits. What we have called artificial bivariate tables (see Figure 12) are made by computing such numbers on the basis of the bivariate normal frequency distribution. The problem is akin to that of determining, in the one-variable case, the proportion of the data between two values, but differs from it in that, because of the number of parameters involved, concise, easily usable tables for constructing artificial bivariate tables do not exist. However, these bivariate tables are easily and quickly computed.*

The artificial bivariate tables have a number of things to recommend them over the conventional tables. The former are much more available than the latter since only the means, standard deviations, and correlation coefficients are required to construct them. The correlation source book of Churchill et al. (1977) contains all the information needed to create artificial bivariate tables for every pair of variables measured in each of the seven surveys covered there--a number of tables well in excess of 50,000. In contrast, few conventional bivariate tables have been published for these data. Some 500 such tables appear in Anthropometry of Air Force Women by Clauser et al. (1972) and about 100 in a report of the 1946 survey of women separating from the U.S. Army (Randall and Munroe, 1949). Conventional tables for all pairs of variables can, of course, be computed from the raw data, but this requires access to these data plus considerably more computer time than do the artificial bivariates.

Since the artificial bivariate tables are based on the summary statistics rather than the raw data, they are independent of the units in which the data were measured. For example, instead of a stature-weight table with stature in centimeters and weight in pounds--a combination of units pleasing to nobody but almost universal for U.S. data--an artificial bivariate table can be in inches and pounds or in centimeters and kilograms or in any mul-

*See previous footnote about the availability of appropriate programs.

BUTTOCK-KNEE LENGTH

BUTTOCK-POPLITEAL LENGTH	20"–21"	21"–22"	22"–23"	23"–24"	24"–25"	25"–26"	26"–27"	27"–28"	Total
23"–22"					1	8	6	1	16
22"–21"				4	42	53	10		109
21"–20"			5	89	170	43	1		308
20"–19"		3	72	205	76	3			359
19"–18"		21	94	53	4				172
18"–17"	3	16	13	1					33
17"–16"	1	2							3
Total	4	42	184	352	293	107	17	1	1000

Figure 12. Artificial bivariate table for buttock-knee and buttock-popliteal lengths (USAF'67 data).

tiple or fraction of these units, with any derived choice of intervals, and with whatever total frequency count is preferred.

A third advantage is that the bivariate normal distribution tends to smooth the data, and the frequencies in adjacent cells form more or less reasonable patterns. Even with survey samples of several thousands or more, rather irregular patterns which make no biological sense often occur, particularly around the perimeter of a conventional table. The regularities of the artificial bivariate tables can, however, also gloss over irregularities which are real and which do make biological sense. In the use of these tables we must take into account the possibility of this occurrence. This often happens when one of the pairs of variables is weight because of the asymmetry of its distribution. More serious problems may arise when using data based on a group which consists of two or more subgroups which have major anthropometric differences. It would be unwise, for example, to compute an artificial bivariate table for weight and stature for a group consisting of equal numbers of basic trainees and of senior officers, or a table of sitting heights and crotch heights for a group consisting of substantially equal numbers of Blacks, Whites, and Orientals. If such tables are needed, it might make considerable sense to create artificial bivariate tables separately for each subgroup and then combine the computed frequencies. The use of our model, and the computer, makes the creation of such tables a simple matter.

Proportions Disaccommodated by Two-Variable Designs

Equipment and workspace units are frequently designed to functionally fit all potential users except for a small number of individuals who are too small (or too large) in one or the other or both of two bodily dimensions. The number of potential users who will be disaccommodated by such a design depends not only on the number who are disaccommodated on each of the design dimensions but also on the correlation between these dimensions. If, for example, 10% of the potential users are disaccommodated on each dimension, the total proportion disaccommodated can be as low as 10% or as high as 20%. By one more use of our bivariate mathematical model, we can estimate the proportion of potential users who will be left out by a specified design. We can equally well determine pairs of design limits which will exclude a specified proportion of the potential users. (See Figure 13)

The shaded areas in Figure 13 illustrate six different types of design patterns in which the disaccomodated potential users will be:

- Type A - individuals who are too small in either dimension.
- Type B - individuals who are too large in either dimension.
- Type C - individuals who are too large in one dimension or too small in the other.
- Type D - individuals who are either too large or too small in either dimension.

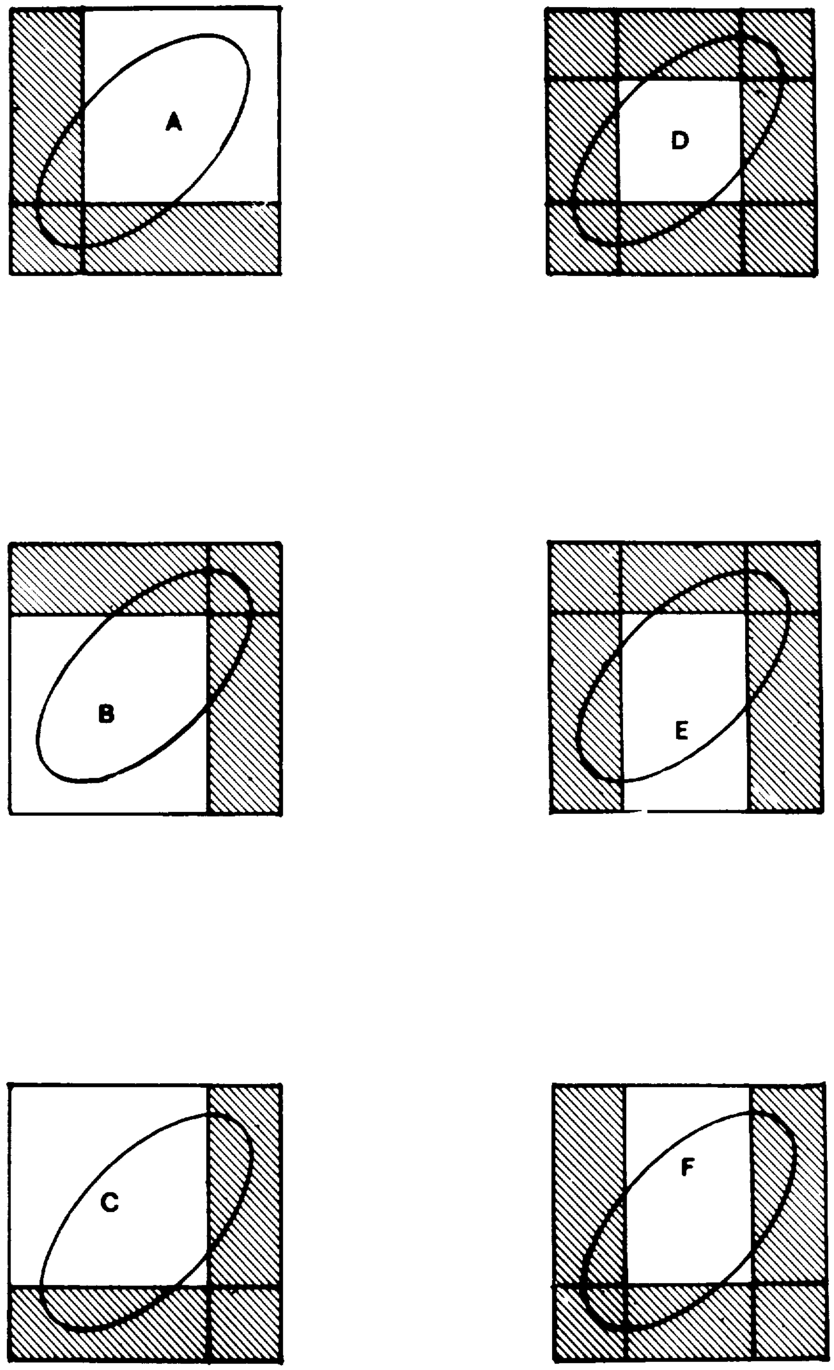

Figure 13. Proportions disaccommodated: six types of two-variable design patterns.

Type E - individuals who are either too large or too small in one dimension or who are too large in the other.

Type F - individuals who are either too large or too small in one dimension or who are too small in the other.

From a statistical point of view, types A and B are equivalent and so are types E and F; we shall, as a consequence, concern ourselves with types A, C, D, and E.

In each case, the problem is the number of persons in the corner box or boxes--those, that is, who are out of range on both dimensions. If the two design dimensions are positively correlated, and we shall assume here that they are, few individuals will be in the lower right-hand corner box or in the upper left-hand one. Type C can, therefore, be easily taken care of. If, for example, a design requires that users not exceed some value in sitting height and that they not fall below some value in arm reach, it will be a rare individual who is out-of-range on both variables. In this case, then, the proportion of potential users disaccommodated will be essentially just the number disaccommodated on sitting height plus the number disaccommodated on arm reach.

For type A designs, the number of individuals in the left-hand column of Figure 12, P_y, and the number in the row at the base of the figure, P_x, are, we presume, either known or can be estimated using Table 2. The number in the corner, P_{xy}, usually must be calculated using a computer program such as the artificial bivariate table or one similar to it. The total number disaccommodated will be given by

$$P_{total} = P_x + P_y - P_{xy} .$$

The eye height, sitting, values for USAF flyers are M = 31.87, SD = 1.19; the design limit of 30.0" is thus (31.87-30.0)/1.19 or 1.57 standard deviations below the mean. From Table 2, we estimate that 5.8% of the flyers fall below the prescribed limit.

Similarly, the thumb-tip reach values are M=31.62, SD=1.57; the design limit of 29.5" is thus (31.62-29.5)/1.57 or 1.35 standard deviations below the mean. Again from Table 2, we obtain 8.9% as an estimate of the proportion of flyers with arms that are too short. The value of P_{xy}, unfortunately, is not so easily obtained since an appropriate table for determining it is not only not available, but would be cumbersome to use if it were. However, by specifying 1.57, 1.35, 0.392 (the correlation between the two dimensions) in the proper computer program, we find that P_{xy} = 1.6%, and,

$$\text{Total disaccommodated} = P_x + P_y - P_{xy} = 5.8\% + 8.9\% - 1.6\% = 13.1\%$$

The type D design can be considered, with reasonable accuracy, as a combined type A and type B design. The number disaccommodated will be essentially the sum of those who exceed one or the other (or both) of the upper limits and of those who fall below one or the other (or both) of the lower limits; the error in this computation will be represented by the rare individuals who are above the upper limit in one dimension and below the lower limit on the other.

Finally, similar reasoning suggests that a type E design be treated as a type B design, plus the strip at the left of the box.

The reverse problem, that of selecting design limits to provide a specified level of accomodation, while more complex mathematically, is easily handled on a computer. Figure 14 shows the output of one computer program which provides for a Type A design, 5%, 10% and 20% disaccomodated design limits for eye height, sitting and thumb tip reach. Any pair of values from the proper curve will serve as appropriate design limits. Thus, for example, a design which will accomodate men who are not over 33.5" in eye height, sitting or over 34.8" in thumb-tip reach is likely to disaccomodate about 10% of the USAF flying personnel. The same will be true of designs based on eye height, sitting and thumb-tip reaches of 34.0" and 33.9", of 35.0" and 33.7", of 35.5" and 33.6", etc.

The problem of selecting rectangular design limits which contain a specified proportion of a set of data can be handled by using the computer program which created Figure 14 twice. An initial run of this program would supply a choice of lower limits which would exclude K_1% of the data, and a second computer run would provide a choice of upper limits designed to exclude K_2 % of the data. The rectangle defined by any pair of lower limits and any pair of upper limits will include at least $(100-K_1-K_2)$% of the data. While the values of K_1 and K_2 may be equal in many problems, nothing in this approach requires that they be equal.

It may be appropriate to end this discussion of the bivariate normal model with the explicit recognition that it is the modern computer which has made this model the useful tool it is. Without a computer, most applications of this model would require the awkward and laborious use of cumbersome tables; what the computer will do well in seconds, it would take hours to do poorly without it.

Sampling Errors

In 1967 the Air Force measured a sample of 2,420 men. Had this survey been carried out a few months earlier or a year later or had a different choice of air bases been made, the sample would have been made up of a somewhat different group of men, a little taller, perhaps, or a bit shorter, somewhat heavier or somewhat lighter. Like the sample that was measured, this hypothetical group would differ in a multitude of ways from the total USAF flying personnel population.

All data collected on samples are subject to sampling error and we cannot, therefore, expect them to represent the population precisely. Parenthetically, we may observe that complete precision is impossible even with 100% sampling; by the time data from a 100% survey of Air Force pilots could be analyzed, and long before such data would be used, the population of

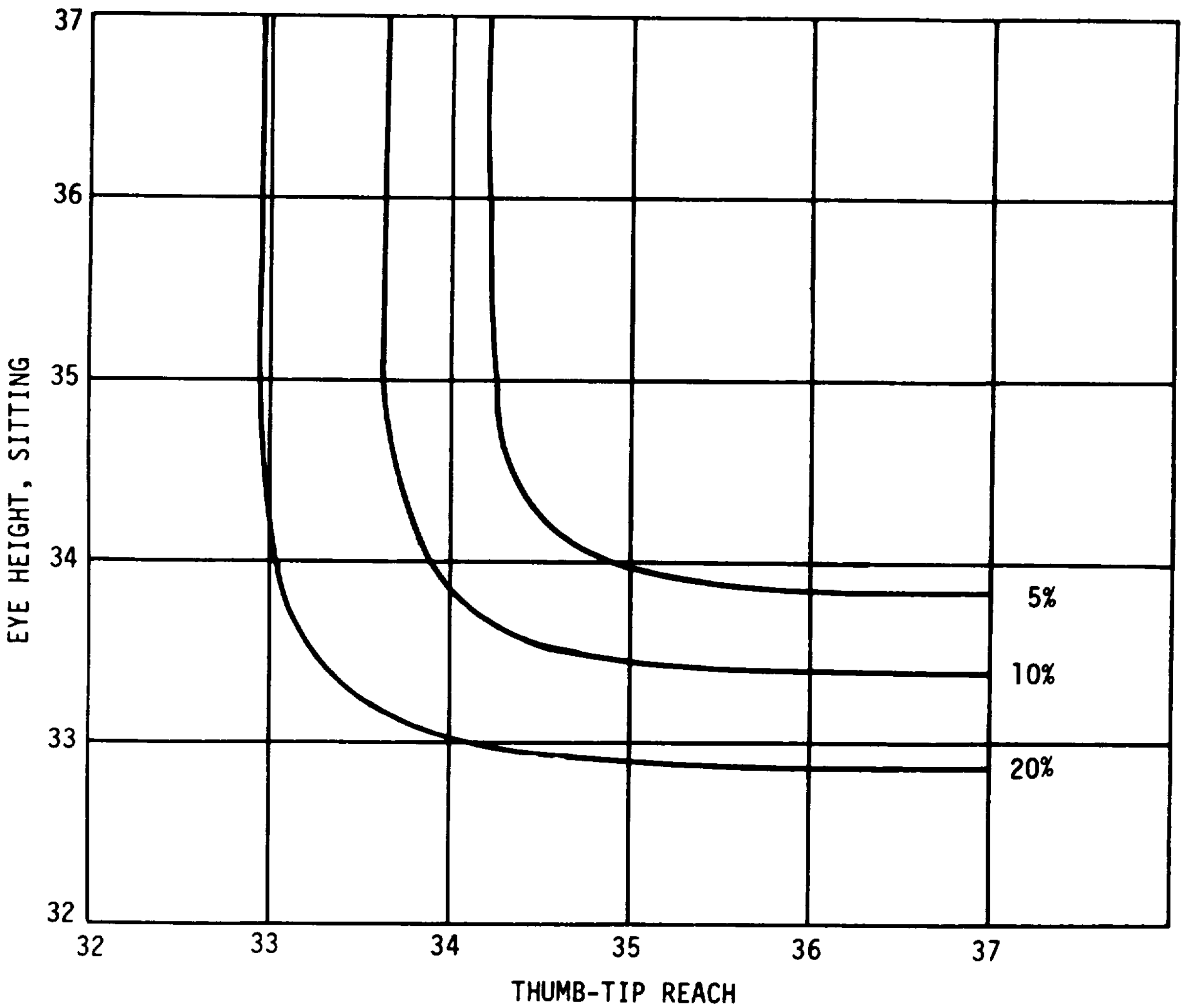

Figure 14. Design limits based on a specified percent disaccommodated: Type A design, Eye Height, Sitting and Thumb-tip Reach.

pilots would already have changed. In addition, the population the design engineer wishes our statistics to describe is usually a population which does not yet exist--pilots or astronauts who will fly or use or wear equipment still years from the production line. Nonetheless, reasonably accurate, useful data can be obtained from samples; this handbook is based on just such data.

Sampling errors arise in the anthropometric surveys from a number of sources; the magnitude of errors from each source is, in the main, all but impossible to evaluate. Military body size surveys are not based on random or "probability" sampling, types of sampling which yield solid estimates of the sampling errors. On the other hand, conscious efforts are usually made to sample a broad spectrum of the population--senior pilots and student navigators, crew members of huge, multi-engine cargo planes and those of much smaller fighters, and so forth. Usually, too, data from various segments of the population can be studied to determine whether significant differences exist among these segments. If important differences are found, separate statistics can be provided for individual subgroups and the statistics for the total group can be adjusted to compensate for sample-population differences in background variables. Various aspects of sample-population matching and of sampling errors are discussed at length in Sampling and Data Gathering Strategies for Future USAF Anthropometry, by Churchill and McConville (1976).

One important component of sampling error is that due to the more-or-less random aspect of the sampling process. This component is directly related to sample size and can be easily evaluated in a probability sense. For most sample statistics, one can compute a statistic known as the standard error which depends on the nature of the statistic, the size of the sample, and, as a rule, the variability of the data, and can be written in the form

$$SE_S = K \cdot SD / \sqrt{N}$$

where the value of K depends on the statistic involved. For the mean, K=1.0; for the standard deviation, K=0.707; for the percentiles, K runs from 1.3 for the 50th percentile, to 2.1 for the 5th and 95th percentiles, and 3.7 for the 1st and 99th ones. For combinations of the mean and standard deviation, such as $M + A \cdot SD$, the value of K is $\sqrt{1 + A^2/2}$.

The standard errors of the basic summary statistics are thus directly proportional to the standard deviation* and inversely proportional to the square root of the sample size. Because of the inverse relationship with the sample size, the standard error for any of these statistics can be made as small as desired by increasing the sample size. However, since the sample size enters this formula as its square root, an increase to four times the original sample size is needed to cut the standard error in half and a nine-fold increase is required to reduce it to one-third.

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*Theoretically, the population SD; however, for samples of 50 or more the distribution is trivial.

In Table 9 we have listed for illustrative purposes a few numerical examples: the standard errors of the mean, standard deviation, and a few percentiles for weight, stature, waist circumference, and foot length for samples of 50, 100, 250, 500, 1,000, and 2,000. In preparing this table we have used standard deviations for the USAF '67 flying personnel survey, but the values from most of the other surveys would give about the same results.

The standard error is a standard deviation type statistic. It does not tell us, of course, what the error is in any instance. If it did, we could use this information to convert the computed value into an errorless one. Rather it enables us to establish probability bounds for the random sampling errors: about 2/3 of the time the error will be less than ± 1 SE; about 95% of the time the error will be less then ± 2 SE; almost never will the error be more than ± 3 SE. The term "confidence limit" is given to bands established by adding to and subtracting from a statistic certain multiples of its standard error. The range from 1.96 standard errors below to 1.96 standard errors above a sample statistic constitutes a "95% confidence limit" the range based on 2.58 standard errors, a "99% confidence limit," and so forth.*

From the values in Table 9, it is quite clear that random sampling errors are fairly small for samples of 1,000 or more but can be of considerable size for the small samples--30 or so--which are often used in experimental studies. For example, the 95% confidence limits for the 5th percentile of stature for the USAF '67 flying personnel data are from about 68.8"-0.2" to 68.8"+0.2", while the same confidence limits for a sample of 30 men would extend from about 2" below the sample mean to 2" above it.

Churchill and McConville (1976) have suggested that samples of 350 will usually be large enough to provide design values of adequate accuracy, and that samples of 250 will generally suffice if certain control processes are used. An essential part of their analysis was the selection of criteria for judging what constituted adequate accuracy. Their conclusions about the adequacy of samples of 350 and 250 were based, in part, on the assumption that 5th and 95th percentile values of height measurements need not be known with greater relative precision than the diurnal variations in stature and that circumferential measurements need not be known with greater relative precision than the cyclic variation in chest circumfernece. It is likely that other criteria based on realistic design factors would have led to similar results.

The random sampling errors of two major statistics, the correlation coefficient and the range, do not follow the pattern of the standard errors just discussed. The sampling error for the correlation coefficient, like

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*Appropriate multiples for other confidence limits can be obtained from Table 5. Note, however, that the constant for 90% confidence limits, for example, is that for the 95th percentiles, and not that for the 10th and 90th.

TABLE 9
TYPICAL STANDARD ERRORS

	SAMPLE SIZE					
	50	100	250	500	1000	2000
Weight (lbs)						
Mean	3.03	2.14	1.36	0.96	0.68	0.48
Std. Dev.	2.14	1.51	0.96	0.68	0.48	0.34
5th/95th%ile	6.37	4.50	2.85	2.01	1.42	1.01
1st/99th%ile	11.22	7.93	5.02	3.56	2.51	1.78
Stature (cm)						
Mean	0.35	0.24	0.15	0.11	0.08	0.05
Std. Dev.	0.24	0.17	0.11	0.08	0.05	0.04
5th/95th%ile	0.72	0.51	0.32	0.22	0.16	0.11
1st/99th%ile	1.28	0.90	0.57	0.41	0.29	0.20
Waist Circumference (cm)						
Mean	0.41	0.29	0.18	0.13	0.09	0.07
Std. Dev.	0.29	0.21	0.13	0.09	0.07	0.05
5th/95th%ile	0.86	0.61	0.39	0.27	0.19	0.14
1st/99th%ile	1.52	1.08	0.68	0.48	0.34	0.24
Foot Length (cm)						
Mean	0.07	0.05	0.03	0.02	0.01	0.01
Std. Dev.	0.05	0.03	0.02	0.01	0.01	0.01
5th/95th%ile	0.14	0.10	0.06	0.04	0.03	0.02
1st/99th%ile	0.25	0.18	0.11	0.08	0.05	0.04

Based on the USAF '67 standard deviations: for weight, 21.44 lbs; for stature, 2.44 cm; for waist circumference, 2.91 cm; for foot length, 0.47 cm.

that for the mean, varies inversely with the square root of the sample size, but does not depend on the standard deviation. Rather, this error depends on the correlation itself in a somewhat complex fashion. The confidence limits do not as a rule extend equally above and below the sample correlation coefficient. This nonsymmetry is minor for samples of several thousand, but becomes substantial when the sample size is small and the correlation coefficient large. Tables and formulas for determining confidence limits for correlation coefficients are given by Churchill et al. (1977). The following values illustrate the 95% confidence limits for correlation coefficients for the USAF '67 flying personnel and 1968 Air Force Women's surveys:

r	AFW '68	FLY '67	r	AFW '68	FLY '67
0.0	±0.045	±0.040	0.70	-.024,+.022	-.021,+.019
0.2	-.004,+.043	-.030,+.029	0.75	-.020,+.018	-.018,+.016
0.3	-.041,+.040	-.028,+.028	0.80	-.017,+.015	-.017,+.013
0.4	-.038,+.037	-.034,+.033	0.85	-.013,+.011	-.011,+.010
0.5	-.034,+.032	-.030,+.029	0.90	-.009,+.007	-.008,+.006
0.6	-.030,+.027	-.026,+.024	0.95	-.005,+.003	-.004,+.003

Thus, for example, the 95% confidence limits for a correlation coefficient whose sample value was 0.600 would be from 0.570 to 0.627, if based on a sample of 1,905, or from 0.574 to 0.624 if based on a sample of 2,420. Ninety-nine percent limits in these cases would be from 0.561 to 0.636 and from 0.565 to 0.633.

The sample range has a standard error which decreases quite slowly as the sample size increases, much more slowly than, for example, the standard error of the mean. The large standard error which the range has, even for rather large samples, is one of the many reasons that the range is usually judged a poor statistic for all but the smallest of samples ($N<20$).

One final standard error--that of a proportion--does not appear to follow the basic formula although it is in fact equivalent to the standard error of the mean. For a proportion P, the standard error is

$$SE_P = \sqrt{P \cdot (1-P)/N} \quad .$$

For values of P from 30% to 70%, this error is roughly equal to $0.5/\sqrt{N}$.

The non-random sampling errors present in our data cannot, as a rule, be evaluated by any set of mathematical formulas. Extensive compilations of data, such as that given in Volume II, do provide a basis for some evaluation of these errors. Each user of these data will probably wish to make such evaluations in his own way, and we limit ourselves to a pair of illustrations of one approach.

One source of error is the failure of a sample to properly correspond to the population in terms of background variables. One might question, for example, whether the sample in the USAF '67 survey accurately reflects the division between students and rated officers, or between pilots and navigators, and, if the sample is faulty in either of these respects, to what extent are the body size statistics affected. Because Volume II contains detailed statistics in this case not only for the entire sample but for the four relevant subgroups as well, we can estimate the changes in the total group statistics that shifts in the sample composition would make. On checking the values for stature, for example, we find that only one of the subgroup means--that for the rather small (N=188) group of student navigators--differs from the total group mean by much more than 0.15". From this perhaps we can conclude that, for stature at least, errors of this type in the sample composition are not likely to be significant errors. We might also be willing to assume that the same thing is done for comparable surveys, statistics such as the RAF 2,000 man survey, for which subgroup data are not available.

Differences in measuring techniques are, of course, a major source of differences in the statistical summaries. Unfortunately, differences in measuring techniques may be present even when published descriptions of the techniques agree. Again, the wealth of material in Volume II will often provide a basis for judging the comparability of measurement techniques. One might, for example, compare the statistics of major measurements on the basis of the statures and weights, the statistics for head and face measurements on the basis of head breadths and head lengths, and so forth, in two surveys. The following numbers are the result of one such comparison based on the USAF '67 flying personnel survey data and the data given by Bolton (1973) for the Royal Air Force 2,000-man survey:

	Weight	Stature	Crotch Height	Chest Circ.	Waist Circ.
RAF	165	69.9	33.6	38.3	33.7
USAF	174	69.8	33.5	38.8	33.5

If we can assume the weight and stature means to be correct, we would expect that mean values for other heights would be about the same, but that USAF circumferences would be a bit larger than the RAF ones. The crotch height and chest circumference mean values clearly follow this pattern; those for waist circumference do not. This last result is not surprising, however, since according to the RAF survey report (Bolton et al., 1973) the technique that the British used to measure waist circumference was quite different from that used in the USAF survey.

Comparison of all the statistics for a single dimension will also provide some clues as to the likely reliability of the measurements and the consistency of the measurement techniques. The wide range of mean values for interscye, maximum, for example, would suggest--and correctly so--that this measurement is quite sensitive to small differences in measurement technique and subject position. Much more typical than interscye maximum are the many dimensions for which the statistics show, in the main, only small and logical patterns of differences. The data for these dimensions are ones in which we can put considerable confidence.

Little, Average, and Big Men and Women: 5th, 50th, and 95th Percentiles

With the growing acceptance of the need to design for small and large individuals as well as for those of average size, the practice has arisen of designating small, average and large men and women as 5th percentile, 50th percentile and 95th percentile men or women. To the extent that these terms imply individuals who are 5th, 50th or 95th percentiles in all dimensions*, there are statistical problems with the concept of the "percentile individual." This is particularly true when the concept is applied to anthropometric dummies, head and body forms, and to other items in which a multiplicity of dimensions must be integrated.

The 50th percentile man and woman differs from other percentile men and women in that they are statistically possible, if rather improbable. Actually, even for this to be true we need to equate the 50th percentile with the mean value. We note in Table 10, for example, that the mean value for waist height (39.5") of Air Force women, plus the mean value for the vertical distance from waist level to vertex (24.3") is equal to the mean value of their statures (63.8").

TABLE 10
SELECTED STATISTICS FOR STATURE AND FLOOR-TO-WAIST AND WAIST-TO-VERTEX HEIGHTS (AFW '68 DATA)

	Mean	5%ile	95%ile	X-1.65SD	X+1.65SD
Stature	63.8"	60.0"	67.8"	59.9"	67.6"
Waist height	39.5"	36.6"	42.5"	36.6"	42.4"
Waist to vertex	24.3"	22.7"	26.1"	22.6"	26.0"
SUMS	(63.8")	(59.3")	(68.6")	(59.2")	(68.4")

Stature could be further segmented: the distance from floor to ankle, from ankle to calf, from calf to knee and so forth. No matter how this segmentation is carried out, the sum of the mean values of the segment lengths of the 50th percentile person will inevitably add to the mean value of stature (see Table 11). Similarly, we can expect that a man who is of average value in height and all of his breadths, depths, and circumferences, will also be of average weight.**

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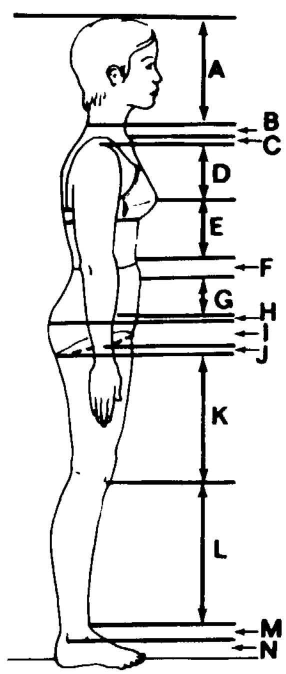
*Hertzberg (1970) defines an anthropometric dummy as one which "...closely approximates a given percentile level of the human body in size, form, segment mobility, total weight, segment weight, (and) weight distribution."

**Even with the mean values there is a minor problem: mean values for indices and shape variables are not as a rule equal to the indices computed from the mean values of the dimensions, but the differences are usually trivial. For example, the mean value for the sitting height/stature ratio (AFW '68 data) is 52.82%; the ratio of mean sitting height (85.60 cm) to mean stature (162.10 cm) is 52.81%.

TABLE 11
FIFTH PERCENTILES, MEANS AND NINETY-FIFTH PERCENTILES FOR STATURE SEGMENTS
(Based on Clauser et al., 1972)



	Stature Segment	5%ile	Mean	95%ile
A	Cervicale Level to Vertex	20.8	22.91	25.1
B	Suprasternale Level to Cervicale Level	5.1	7.19	9.4
C	Acromial Level to Suprasternale Level	- 2.1	0.14	2.4
D	Bustpoint Level to Acromial Level	10.4	13.54	16.8
E	Waist Level to Bustpoint Level	14.3	18.04	21.9
F	Abdominal Extension Level to Waist Level	4.9	7.13	9.7
G	Trochanteric Level to Abdominal Ext. Level	7.1	10.48	13.6
H	Buttock Level to Trochanteric Level	- 2.8	0.46	3.9
I	Crotch Level to Buttock Level	4.7	7.71	10.5
J	Gluteal Furrow Level to Crotch	- 1.3	1.80	5.1
K	Tibiale Level to Gluteal Furrow Level	26.7	30.72	34.8
L	Ankle Level to Tibiale Level	27.4	30.80	34.4
M	Lateral Malleolus Level to Ankle Level	2.4	4.41	6.8
N	Floor to Lateral Malleolus	5.8	6.77	7.8
O	TOTALS	123.4	162.10	202.2
		(48.6")	(63.8")	(79.6")

This, however, is not the case if we consider percentile values other than the 50th. Except for the 50th percentile, no percentile man can possibly exist. The essence of the problem with other "percentile men", the 5th, 95th, and so forth, is that, as illustrated in Table 10, percentiles (other than the 50th) for segments do not add to the corresponding percentile for the sum of the parts. A set of segments, all 5th percentile, will add to less than the 5th percentile for the total; the 95th percentiles of the segments will, in turn, exceed the 95th percentile of the total. The problem is not that a man cannot be 5th percentile in crotch height and sitting height but that if he is, he isn't 5th percentile in stature. In Table 10, we note that the sum of the 5th percentiles for waist height (36.6") and waist level to vertex (22.7") is 59.3". This sum is 0.7" less than the 5th percentile for stature and approximately equal to stature's 2nd percentile. Similarly the two 95th percentiles add to 68.6", a value close to the 98th percentile for stature.

The exact way in which these percentiles have been computed has nothing to do with the heart of the problem nor has our choice of the 5th and the 95th percentiles. We can change our approach slightly and use values of the mean minus 1.65 standard deviations to define our 5th percentile man and values of the mean plus 1.65 standard deviations for the 95th percentile man. But such a change changes little or nothing, as the last two columns of Table 10 show. Just as the percentiles of the parts don't add to the corresponding percentile of the whole, the standard deviations of the parts--here they are 1.77" for waist height, and 1.04" for waist level to vertex--don't add to the standard deviation of the whole--2.36" for stature. The man who is 1.65 standard deviations above average on these two segments of stature simply does not end up 1.65 standard deviations above average in stature.

When we divide stature into more than two segments, the differences are even larger. In Table 11, are the 5th and 95th percentiles and mean values for 14 vertical distances which together constitute stature computed on the same 1968 Air Force Women's data. As was to be expected, the mean values for these segments add exactly to the mean value for stature (162.10 cm or 63.8"); the sums of the percentiles, on the other hand, differ drastically from the corresponding percentiles for stature. The sum of the 5th percentiles is a tiny 48.6"--almost a full foot less than the 5th percentile for stature and about 8" less than the height of the shortest woman measured in the AFW '68 survey. Similarly, the sum of the 95th percentiles, 79.6", is almost a foot above the 95th percentile and about 8" more than the height of the tallest survey subject.

These results are not unique to our choice of illustrative data. Rather they are a direct consequence of the fact that the standard deviation of a sum is given by the formula:*

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*This formula is similar to the one for the length of the 3rd side of a triangle with r being related to the angle between the two known sides; the 3rd side is always less than the sum of the other two except in the abnormal case in which the triangle degenerates into a line segment.

$$SD_{x+y} = \sqrt{(SD_x)^2 + 2r\ SD_x SD_y + (SD_y)^2}$$

where r is the correlation coefficient for X and Y. Only if $r = 1.00$ will $SD_{x+y} = SD_x + SD_y$, otherwise the standard deviation of the sum is less than the sum of the standard deviations of the parts. Since, in practice, the correlation coefficient is always less than 1.00, the results will be similar to our illustrative case. The formula for the sum of three or more parts is similar to that for two parts; the standard deviation of the total will be equal to the sum of the separate standard deviations only if all the relevant correlation coefficients are equal to 1.00.

Not only don't percentile values for a set of linear segments add to the percentile value of the total, but the percentile values for interrelated dimensions in a cross section of the body do not as a rule mesh together and can lead to distorted shapes. Cross section areas of normal shape based on 95th percentile breadths and depths will as a rule exceed the 95th percentile in area; a man consistently 95th percentile in breadths, depths, and heights will unquestionably be abnormally heavy, a man consistently 5th percentile abnormally light.

An analysis of the problems of achieving percentile values for "form" will be omitted here, in part because most statistical measures of form yield ambiguous definitions of large and small. It is not unusual that an individual could be both 5th percentile and 95th percentile on the basis of logically equivalent definitions of the same shape measure. For example, a USAF woman with a head breadth of 6" and a head length of 7" is about 95th percentile on the head breadth-head length (cephalic) index; she is also about 5th percentile on the head length-head breadth index. Traditionally, it is true, anthropologists have divided head breadth by head length, but this practice is quite arbitrary.

This non-existence of all percentile men except the 50th is a problem relating basically to anthropometric dummies and coordinated body forms. Exceedingly useful "large" test dummies, "small" head forms, and the like can, of course, be constructed. The design of such dummies and forms, however, will have to be based on a perceptive awareness of the way the multiplicity of body dimensions interrelate and the statistical principles which describe these relationships. It is, in addition, highly likely that there will need to be an awareness that there are many "types" of large men, and that, for different uses, body forms will need to be based on different designs. Some of the matters relevant to this problem are discussed by McConville and Churchill (1976).

The design of equipment which must accommodate big men or small men is quite different from attempting to create a body form which corresponds to equivalent percentile values in all dimensions. No theoretical limitations usually exist to rule out designs which will simultaneously accommodate men who are 95th percentile in one dimension, others who are 95th percentile in a second dimension, and still others who are 95th percentile in a third dimension. Such designs require only the proper anthropometric data and, all too often, considerable amounts of insight and ingenuity.

The Monte Carlo Method

Some statistical problems which are awkward to solve directly can be tackled by what has come to be known as the Monte Carlo Method. While it has rarely been used in dealing with anthropometric data, we conclude this chapter with a brief description and illustration of this method in the hope that its potential value in helping with design problems will not be ignored.

The typical Monte Carlo type problem is one which asks the question: If we do something by some random method, what are the probabilities of some particular set of outcomes? The essence of the method is to actually "do" the same thing a great many times, usually with the aid of random numbers and a computer, and observe the outcomes. The relative frequency of each outcome is then considered as an approximation to its probability. Such an approach can be used with problems so complex that few alternative approaches exist; it is also a practical solution to many less complex problems for which feasible, but laborious or time consuming alternatives, are available.

As an illustration of the Monte Carlo method, we consider a problem of picking crews consisting of five members each. The members of each crew are to be selected randomly from a population similar to the USAF 1967 flying personnel group with the restriction that no man over 5'8" tall or weighing more than 165 lbs will be accepted. The questions to be answered are:

(1) What is the distribution of total crew weights? and
(2) What is the distribution of the maximum stature in each crew?

To answer these questions, the statures and weights of the 2,420 subjects in the 1967 survey sample were stored in the computer. By the use of random numbers, subjects were selected until five satisfying the height and weight limitations were obtained. The total weight of these men was then computed and the height of the tallest man noted. The process was continued until samples of 100, 200, 500, and 1,000 "crews" were obtained. The results are summarized in Tables 12 and 13.

In using this method, we presumably select samples until the results show a stable pattern. The entries in Tables 12 and 13 for 1,000 trials are not very different from those for 500 trials. In fact, the general distribution suggested by the results of 100 trials is rather similar to that for 1,000. Median values, for example, for total crew weight and maximum stature, are about 752 pounds and 172.1 cm (67.8") for each number of trials.

The procedure just described was based on using actual survey data, but the availability of these data is not essential. We could have had recourse once again to the bivariate normal distribution as a mathematical model for the statures and weights, constructed an approximation to the stature-weight distribution,* and sampled it just as we sampled the actual

*This could be done in a variety of ways. One method, for example, could be based on the artificial bivariate program. A second method could be based on repeated selection of pairs of uncorrelated, normally distributed, random

TABLE 12
DISTRIBUTION OF WEIGHTS OF FIVE-MAN CREWS*

	NO. OF TRIALS			
	100	200	500	1000
Weight (lb)				
800 & up		0.5%	1.2%	1.0%
790 & up	2.0%	2.5%	3.6%	3.4%
780 & up	12.0%	11.5%	12.2%	11.6%
770 & up	22.0%	21.0%	22.0%	20.5%
760 & up	42.0%	39.5%	40.2%	35.9%
750 & up	55.0%	55.0%	55.4%	53.7%
740 & up	67.0%	67.0%	67.4%	68.3%
730 & up	78.0%	78.5%	79.2%	80.3%
720 & up	84.0%	85.5%	87.4%	88.9%
710 & up	87.0%	91.0%	93.6%	94.2%
700 & up	91.0%	94.5%	96.8%	97.3%
690 & up	97.0%	99.0%	99.2%	99.5%
680 & up	98.0%	99.5%	99.6%	99.8%

* See text for method of selection.

TABLE 13
DISTRIBUTION OF MAXIMUM STATURES OF FIVE-MAN CREWS*

	NO. OF TRIALS			
	100	200	500	1000
Stature (cm)				
172.6 & up	15.0%	12.5%	10.2%	10.0%
172.4 & up	30.0%	30.0%	28.8%	27.1%
172.2 & up	48.0%	50.0%	46.0%	44.2%
172.0 & up	68.0%	67.5%	63.8%	61.3%
171.8 & up	75.0%	75.5%	73.8%	73.2%
171.6 & up	80.0%	79.5%	79.0%	79.4%
171.4 & up	88.0%	87.5%	85.2%	84.3%
171.2 & up	88.0%	88.5%	86.4%	86.2%
171.0 & up	92.0%	92.0%	90.2%	89.9%
170.8 & up	94.0%	94.0%	92.4%	92.5%
170.6 & up	94.0%	94.5%	93.6%	94.0%
170.4 & up	97.0%	96.5%	95.6%	96.2%
170.2 & up	97.0%	97.0%	96.6%	97.3%

* See text for method of selection.

distribution. To do this, only the means and standard deviations plus the correlation coefficient are needed.

numbers, X_1 and X_2 and then setting: stature = X_1 and weight = $(X_2 - rX_1)/\sqrt{1-r^2}$, where stature and weight are interpreted as being in standard deviation units, and r is the correlation coefficient.

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